A Truthful QoS-Aware Spectrum Auction with Spatial Reuse for Large-Scale Networks

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Abstract—In cognitive radio networks (CRNs), a wireless user with primary access right on a channel (called *primary user*) has prioritized access to the channel and the user with secondary access right (called *secondary user*) can use the channel when the primary user is idle. Spectrum auction has emerged as a promising approach to address the access allocation problem in CRNs. A significant challenge in designing such auction is providing truthfulness to avoid market manipulation. In most previous work, the primary access rights on channels are pre-determined before the auction and bidders can only compete for the secondary access rights. However, a user's requirement on spectrum access rights relies on their QoS demands. Therefore, it is much desirable to allocate spectrum access rights on the basis of QoS demands as well as to exploit the resulting spatial spectrum reuse opportunities. To solve this problem, we propose TRUMP, a truthful spectrum auction mechanism, by taking into consideration both QoS demands and spectrum spatial reuse, which can drastically improve spectrum utilization. The theoretical analysis proves that TRUMP achieves *truthfulness* and *individual rationality* with polynomial-time complexity. Our extensive simulation results show that our proposals outperform previous work in terms of both social welfare and spectrum utilization.

Index Terms—Cognitive Radio Networks, Spectrum Auctions, Truthfulness, Algorithms.

1 Introduction

Radio spectrum is a critical and scarce resource for wireless communications. Traditionally, it is regulated by spectrum regulators (*e.g.,* Federal Communications Commission in USA) and allocated to users by assigning exclusive licenses. Only users with licenses are allowed to use the licensed spectrum. In recent years, with the proliferation of novel wireless technologies, the demand for radio spectrum is drastically increased. However, investigations show that most licensed spectrum is vastly under-utilized while unlicensed portion is increasingly overcrowded under such kind of static and exclusive allocation policy [1].

Cognitive Radio Network (CRN) [2], [3] is emerging as a promising solution to overcome the above dilemma. In CRNs, the access rights of channels are divided into two categories: primary access right and secondary access right. A wireless user with primary access right on a channel (called *primary user*) has prioritized access to the channel and the user with secondary access right (called *secondary user*) can only use the channel when the primary user is idle. In addition, the secondary user must vacate the channel as soon as the primary user initiates its transmission activity. CRNs provide a new venue to exploit the under-utilized portion of the spectrum band while limiting the interference imposed on primary users. The crucial problem faced by the spectrum regulator is how to select the users on the access rights of each channel so as to optimize the spectrum utilization. Generally, there are two kinds of allocation schemes, named the *two-step allocation* [7] and the *single-step allocation* [9] respectively. In the former schemes, the regulator first assigns spectrum to primary users in a static manner and let the primary users allocate their idle portion of spectrum to secondary users independently. In the latter schemes, the regulator dynamically determines either the primary or secondary access rights on spectrum to users within one step.

After extensive research during the past five years, the auction mechanisms for two-step allocation (referred as secondary spectrum auctions, SSAs) [4], [5], [6], [7], [8] have been relatively well understood. In SSAs, primary users are pre-determined before starting the auction and bidders can only compete for secondary access rights on channels. In fact, a user's requirement on spectrum access rights relies on the QoS demands of his applications and thus may vary over time. For instance, when a user runs time-sensitive applications such as live streaming, it would like to request for primary access right with higher price. On the other hand, when performing delay-tolerant applications like file transfer, it may prefer secondary access right with lower price. Therefore, it is appealing to assign the access rights in accordance with users' QoS demands. Under such considerations, single-step allocation is much more desirable. It enables the regulator to flexibly assign access rights to users on-the-fly according to their QoS demands. In addition, it is also beneficial for users since they can optionally request for different levels of access rights with optimized prices.

According to our best knowledge, [9] is the only work

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designed for this scenario till date. The authors propose an auction framework to allow bidders to bid for primary and secondary access rights in accordance with their traffic model. They present truthful auctions by using the Vickery-Clarke-Groves (VCG) mechanism [11], a generally well-known incentive compatible mechanism, to maximize the social welfare of networks. Although the VCG mechanism is a possible choice enforcing that users bid their true valuations, it is also well known to suffer from several drawbacks such as low revenue [12], [13]. Moreover, the spatial reuse of spectrum has not been exploited. It has been widely accepted that the spatial reusability of spectrum can drastically improve spectrum utilization by addressing the interference constraints [6]. Unfortunately, finding an interference-free spectrum allocation is NP-hard [10]. As a result, it becomes more challenging to guarantee the truthfulness of such spectrum auction. Firstly, it is well-known that the truthfulness of VCG mechanism is achievable for only optimal solution [20]. Secondly, the spatial reusability makes traditional auction mechanisms, *e.g.*, k-position [21], lose truthfulness. This is because multiple well-separated buyers can use the same spectrum band simultaneously, which constitutes the fundamental difference between spectrum and traditional goods (*e.g.*, paintings). Finally, earlier SSAs with interference constraints are unsuitable for single-step allocation since a bidder in SSAs has only two states (win or lose) while three states exist (primary, secondary and lose) in the latter.

In this paper, we investigate the single-step allocation problem and propose TRUMP, a **TRU**thful spectru**M** auction considering QoS demands and s**P**atial reuse. TRUMP dynamically assigns spectrum access rights to users according to their QoS demands. TRUMP provides a bidding language to allow users to independently express bids for their valuations on spectrum in terms of QoS demands. To exploit the spectrum reusability, we formulate the QoS-aware spectrum access allocation with interference constraints as an integer programming problem where the social welfare could be maximized. While finding such optimal solution is shown to be NP-hard, we focus on computationally efficient spectrum allocation and pricing algorithms to support large scale networks. We first propose TRUMP for the *single-secondary-user* case, where each channel is allocated to one primary user and at most one secondary user. TRUMP consists of a computational-efficient allocation algorithm and a finely designed pricing scheme. Following that, we extend it to r-TRUMP for *multiple-secondary-user* case, where one primary user and at most r secondary users can be allocated to each channel. Both TRUMP and r-TRUMP are shown to be truthful and computationally efficient. We evaluate the performance of our proposals by extensive simulations. Simulation results show that TRUMP can effectively and efficiently improve both social welfare and spectrum utilization compared with prior work. To the best of our knowledge, we are the first to exploit the spatial spectrum reusability for single-step allocation. The main contributions of this paper are as follows:

- We present a QoS-aware auction framework for single-step allocation. A novel bidding language is provided for bidders to dynamically bid for primary or secondary access rights according to their QoS demands.
- We design practical and efficient auction mechanisms that are simple and scalable, yet provide powerful performance guarantee. Spectrum utilization are drastically improved by taking into account spectrum reusability.
- The proposed mechanisms are shown to be *truthful* and *individually rational*. The former guarantees no additional profit could be awarded for bidders by cheating. The latter ensures that the profit of each bidder is no less than zero, which incentivizes bidders to voluntarily participate the auction.

The rest of the paper is organized as follows. Section II introduces the system model. Section III gives the problem statement. Sections IV and V presents the auction design of TRUMP and its extension r-TRUMP, respectively. Performance evaluation is shown in section VI. Section VII concludes the paper. Related work and analysis of truthfulness and individual rationality are given in the supplementary file.

2 System Model

2.1 QoS-aware Auction

In heterogeneous multiuser networks, QoS demands may vary from user to user. Without loss of generality, we consider two categories of services: QoS-guaranteed and best-effort services [16], [17]. The former is provided either with sufficient resources such that the QoS requirement is guaranteed, or with no resources at all. The latter, with low priority, is served through the network's best-effort. In our QoS-aware spectrum allocation scheme, primary access rights are suitable for QoS-guaranteed services like online-video, while secondary access rights are suitable for best-effort services like FTP and HTTP.

Accordingly, we consider these two types of bidders as follows using a tuple bidding language $b_i = \langle b_i^p, b_i^s \rangle$, where b_i^p and b_i^s represent the bid issued by bidder i for primary and secondary access, respectively.

- **TYPE-I:** Bidders with QoS-guaranteed services who only accept primary access on channels. Since the secondary access right is unacceptable, it is reasonable to set $b_i^p > 0$ and $b_i^s = -\infty$ for any bidder i of TYPE-I.
- **TYPE-II:** Bidders with best-effort services who would accept either primary or secondary access on channels, but primary access is much preferable. These can interpreted as $b_i^p \geq b_i^s > 0$ for any bidder i of TYPE-II.

Note that these settings are based on different valuations for primary and secondary access. In a similar way, we use $v_i = \langle v_i^p,\, v_i^s \rangle$ to denote the *true valuation* of bidder i for primary and secondary access, respectively.

An auction consists of an allocation algorithm along with a pricing scheme. All bidders simultaneously submit their bids privately to the regulator without any knowledge of others. After collecting bids, the auctioneer determines the winners of primary and secondary users according to the allocation scheme, and a price τ_i will be charged from each winner i accordingly. The utility u_i (*i.e.*, profit) of bidder i is computed as follows.

$$u_i = \begin{cases} v_i^p - \tau_i, & \text{if } i \text{ wins primary access,} \\ v_i^s - \tau_i, & \text{if } i \text{ wins secondary access,} \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

2.2 Network Model

We consider a spectrum market with M orthogonal channels denoted as $\mathcal{M} = \{1, 2, \cdots, M\}$ and N buyers (or bidders) denoted as $\mathcal{N} = \{1, 2, \cdots, N\}$. The auction is conducted by the spectrum regulator (called auctioneer) and we assume the bidders do no collude. To avoid transmission conflicts on the same channel, the *protocol interference model* [14], [15] has been widely adopted to formulate the impact of interference for resource allocation problems. Interference information can be obtained from statistics collected during operation or extensive spectral measurement (*e.g.*, on the packet level seek) [19], which is the basic assumption underlying the general model of spectrum auctions [6], [7], [8].

Under such model, the interference can be well captured by a conflict graph $G(\mathcal{N}, \mathcal{E})$, where $\mathcal{E}$ is a collection of $L = |\mathcal{E}|$ edges. An edge $(i, j) \in \mathcal{E}$ between any two bidders i and j ($i, j \in \mathcal{N}$) indicates that they interfere with each other when using the same channel simultaneously. In SSA-based auction mechanisms, bidders i and j will be assigned different channels if both win.

Different from the traditional SSA-based scheme, our QoS-aware auction allows the following two basic allocation types A1 and A2 by differentiating bidders for their various service demands:

- **A1:** A channel is allocated to only one bidder as primary. All its neighboring bidders in the conflict graph must not be assigned the same channel.
- **A2:** A channel is allocated to a pair of bidders which are connected in the conflict graph, one as primary and the other as secondary. All their neighboring bidders in the conflict graph will not be assigned the same channel.

Note that the same channel can be reused for multiple basic allocations if they are apart from each other, *i.e.*, they are not connected in the conflict graph. Therefore, our proposal improves the spectrum reusability because of A2 compared to the traditional ones. In formal, we use notation $a \overset{c}{\leftrightarrow} a'$ to indicate that two different basic allocations a and a' ($a \neq a'$) are connected because of sharing a common node or being attached by an edge in the conflict graph. In the following, we use a node and an edge in the conflict graph to indicate the basic allocation of types A1 and A2, respectively.

3 Problem Statement

3.1 Design Goals

A common goal of designing the allocation scheme is to maximize *social welfare*, *i.e.*, the sum of all winners' bids, which is a strong indicator of how well positioned the buyer is to make good use of the sold spectrum bands.

In auction design, *truthfulness* (or *incentive-compatibility*) is of supreme importance because it can avoid the manipulation that a bidder may be selfish by declaring a false bid instead of his own *true valuation* (*i.e.*,$b_i^p \neq v_i^p$ or $b_i^s \neq v_i^s$) so as to increase his profit. An auction is truthful if each bidder always maximizes his profit by bidding with his true valuation, *i.e.*, $u_i(v_i, \mathbf{b_{-i}}) \geq u_i(b_i, \mathbf{b_{-i}})$ for any b_i when fixing $\mathbf{b_{-i}}$, where $\mathbf{b_{-i}} = (b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_N)$ is the list of bidding prices without i's bid. Therefore, truthful auction makes the bidding easier for bidders. Otherwise, each bidder has to figure out the bidding strategies of others before making an optimal bidding decision for himself.

Another economic property, *individual rationality*, is also required to be satisfied in auction. An auction is of individual rationality if no winner is paid more than its bid. This property guarantees non-negative profit for bidders who bid truthfully, and thus provides them with the incentives for the participation.

3.2 Problem Formulation

In this section, we develop an optimization model for the social welfare maximization problem. We here consider the single-secondary-user case. First, we define binary variables $x_i^p(m)$ and $x_i^s(j, m)$ as follows to characterize the two basic allocation types A1 and A2, respectively, where $i \in \mathcal{N}$, $(i, j) \in \mathcal{E}$, and $m \in \mathcal{M}$.

$$x_i^p(m) = \begin{cases} 1, & \text{if } i \text{ is allocated channel } m \text{ as primary,} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

$$x_i^s(j, m) = \begin{cases} 1, & \text{if } i \text{ and } j \text{ are allocated channel } m \text{ as seconary and primary, respectively,} \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

The interference constraints in our proposal are therefore to guarantee that any two basic allocations, *e.g.*, a and a', should not be assigned the same channel if they are connected, *i.e.*, $a \overset{c}{\leftrightarrow} a'$. All possible combinations are: both of type A1 (node-node case), both of type A2 (edge-edge case), and one of type A1 and the other of type A2 (node-edge case), as formulated in constraints (4),(5) and (6), respectively.

These constraints are explained as follows by Fig. 1, where any dotted line in the conflict graph indicates that

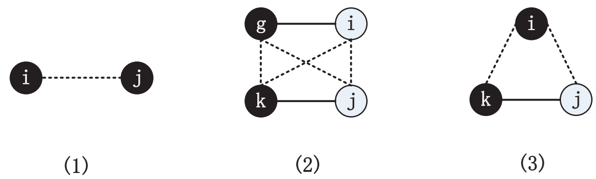


Fig. 1: Constraint illustration for (1) node-node case, (2) edge-edge case and (3) node-edge case. Black and white nodes represent winning bidders as primary and secondary, respectively. Solid lines represent edges in the conflict graph.

the two corresponding basic allocations in constraints (4), (5) and (6) are connected. In other words, they should not be assigned the same channel. For example, constraint (4) for the node-node case describes that a conflict will occur when the primary access right is assigned to both i and j if $i \overset{c}{\leftrightarrow} j$, i.e., $(i,j) \in \mathcal{E}$. In fact, this condition is very similar to the interference in SSA. Constraint (5) for the edge-edge case shows that the connected basic allocations (i,g) and (j,k) will lead to conflict if they share the same channel. Constraint (6) defines the conflict condition for node-edge case in a similar way.

$$
\begin{cases}
x_i^p(m) + x_j^p(m) \le 1, \forall m \in \mathcal{M}, \forall i,j \in \mathcal{N} : i \overset{c}{\leftrightarrow} j & (4) \\
x_i^s(g,m) + x_j^s(k,m) \le 1, & \\
\qquad \forall m \in \mathcal{M}, \forall (i,g),(j,k) \in \mathcal{E} : (i,g) \overset{c}{\leftrightarrow} (j,k) & (5) \\
x_i^p(m) + x_j^s(k,m) \le 1, & \\
\qquad \forall m \in \mathcal{M}, \forall i \in \mathcal{N}, \forall (j,k) \in \mathcal{E} : i \overset{c}{\leftrightarrow} (j,k) & (6)
\end{cases}
$$

Furthermore, each bidder requests only one channel and each channel can be allocated to at most one secondary bidder. These can be formulated by the following constraints.

$$
\begin{cases}
\sum_{m \in \mathcal{M}} \sum_{j:(i,j) \in \mathcal{E}} x_i^p(m) + x_i^s(j,m) \le 1, \forall i \in \mathcal{N} & (7) \\
\sum_{i:(i,j) \in \mathcal{E}} x_i^s(j,m) \le 1, \forall j \in \mathcal{N}, \forall m \in \mathcal{M} & (8)
\end{cases}
$$

Finally, the maximum social welfare problem can be formulated as the following integer programming:

$$
\max \sum_{m \in \mathcal{M}} \sum_{i \in \mathcal{N}} b_i^p x_i^p(m) + \sum_{m \in \mathcal{M}} \sum_{(i,j) \in \mathcal{E}} b_i^s x_i^s(j,m) \qquad (9)
$$
$$
s.t. (4) - (8)
$$

3.3 Complexity Analysis

***Theorem** 1:* The problem of maximizing social welfare in this setting is NP-hard.

Proof: Consider a simple case where all bidders are of TYPE-I, *i.e.*, $x_i^s(j,m) = 0$ for all $(i,j) \in \mathcal{E}$ and $m \in \mathcal{M}$. Then the problem is equivalent to the well studied graph colouring problem, where channel is equivalent to 'color' and bidder is equivalent to 'node'. It is known that the graph colouring problem is NP-hard. □

With the optimal solution, we can get a truthful auction by applying VCG mechanism directly. However, to solve (9) with optimal solution is impractical and non-optimal solution makes VCG mechanism untruthful. On the other hand, the prior truthful SSAs like [6] cannot be applied to our setting, since a bidder has only two states (either win or lose) in SSA while three states exist in our setting (primary, secondary and lose).

TABLE 1: Notations

B	The set of primary bids $\{b_1^p, b_2^p, ..., b_N^p\}$.
W	The set of edge weights $\{w_1, w_2, ..., w_L\}$.
$N(i)$	The set of nodes sharing edge with node i in G.
$Sort(\cdot)$	A function that sorts a set in a descending order.
$Top(\cdot)$	A function that returns the first element in a sorted set.
$Avai(i)$	The set of available channels for node i.
$State(i)$	The state of node i.
$Remove$ (X, i)	If $i \in \mathcal{N}$, it removes the bid of node i and all associated edges $(i,j) \in \mathcal{E}$ from set X. Otherwise, it recursively invokes $Remove(i_1)$ and $Remove(i_2)$, where i_1 and i_2 are the two endpoints of edge i.
$PriceTable$ (i,j,k,m)	A function for charging i to compete channel m, where edge (j,k) is a neighbor of i.

4 TRUMP Auction Design

Before presenting the allocation algorithm TRUMP-Allocation and the pricing algorithm TRUMP-Pricing, we first define the weight of edge $i \in \mathcal{E}$ that joins nodes i_1 and i_2 as:

$$
w_i = \max(b_{i_1}^p + b_{i_2}^s, b_{i_1}^s + b_{i_2}^p). \qquad (10)
$$

The interpretation of weights w_i is as follows. If i_1 and i_2 are primary and secondary bidders on a channel, respectively, the sum of their bids is $b_{i_1}^p + b_{i_2}^s$. So w_i is the maximum sum of bids if i_1 and i_2, one as primary and the other as secondary, are assigned the same channel. Note that, we use the term *node* and *bidder* interchangeably. Other symbols and notations are summarized in Table 1.

4.1 TRUMP-Allocation

Based on the sorted set $B^{'} = Sort(B \cup W)$, TRUMP-Allocation allocates channels to edges or nodes sequentially from the highest weight or bid to the lowest one. Note that an element i in $B^{'}$ can be either a node or an edge. If i is a node, the algorithm first checks whether there are enough channels for assigning, *i.e.*, $|Avai(i)| \ge 1$. If so, it assigns a channel with the least index in $Avai(i)$ to i, and removes this channel from each of the available channel sets of i's neighboring nodes. If i is an edge with endpoint i_1 and i_2, the algorithm

Algorithm 1: Procedure $Assign(i)$

if $i \in \mathcal{N}$ **then**
  if $Avai(i) == \phi$ **then**
    return ϕ;
  find $m \in Avai(i)$;
  assign channel m to i;
  $State(i) = P$;
  remove m from $Avai(j)$ foreach $j \in N(i)$;
if $i = (i_1, i_2) \in \mathcal{E}$ **then**
  if $Avai(i_1) \cap Avai(i_2) == \phi$ **then**
    return ϕ;
  find $m \in Avai(i_1) \cap Avai(i_2)$;
  assign channel m to edge i
  if $b^p_{i_1} + b^s_{i_2} \geq b^s_{i_1} + b^p_{i_2}$ **then**
    $State(i_1) = P$ and $State(i_2) = S$;
  else
    $State(i_2) = P$ and $State(i_1) = S$;
  remove m from $Avai(j)$ foreach
  $j \in N(i_1) \cup N(i_2)$;
return m;

Algorithm 2: TRUMP-Allocation(B,W,G)

$B^{'} = Sort(B \cup W)$;
Set $State(i) = null$ for $i \leftarrow 1$ to N;
Set $Avai(i) = \mathcal{M}$ for $i \leftarrow 1$ to N;
while $B^{'} \neq \phi$ **do**
  $i = Top(B^{'})$;
  call procedure $Assign(i)$;
  $Remove(B^{'}, i)$;

first checks whether there is a common available channel of i_1 and i_2, *i.e.*, $|Avai(i_1) \cap Avai(i_2)| \geq 1$. If so, it assigns this channel to the edge and removes it from the available channel sets of both bidder i_1's and bidder i_2's neighboring nodes. There are three possible states after the allocation: P, S and $null$, which indicate the primary access, secondary access and none access assigned to bidder i, respectively. Note that, the initial state of each bidder is $null$, and the initial set of available channels for each node i is the total channels (*i.e.*, $Avai(i) = \mathcal{M}$). TRUMP-Allocation (see Algorithm 2) contains an assignment procedure which is described in Algorithm 1.

4.2 TRUMP-Pricing

To ensure the truthfulness, the charged price of a bidder must be independent with his own bid. Otherwise, the bidder can manipulate his bid to improve the utility. The design philosophy of pricing is that the charged price for each bidder is the minimal bid for seizing one of its neighbors' channel such that the bidder with any bid higher than the price will always win an access right. Note that neighbors includes both neighboring edges and neighboring nodes.

After allocation, we use pricing method to charge each winner. The TRUMP-Pricing algorithm works as follows: for each winner i, it first takes out i and its associated edges from $B^{'}$, and runs TRUMP-Allocation on the updated $B^{'}$ again. Note that variable $Avai()$ needs to be reinitialized. Algorithm 3 describes the routine of TRUMP-Pricing, where MAXNUM represents an infinite positive number and array $d[m]$ records the maximum weight/bid of i's neighbors attached on channel m. Since the weight/bid set ($B^{'}$) is sorted in a descending order, $d[m]$ is actually the first neighbor's weight/bid on m.

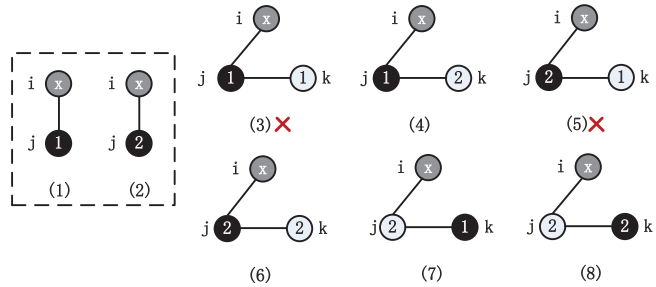


Fig. 2: All combinatorial cases for charging bidder i. Nodes 1 and 2 represent bidder of TYPE-I and TYPE-II, respectively. Black and white nodes represent winning primary access and secondary access in TRUMP-Pricing, respectively. Node x represents a bidder of any type.

Algorithm 3: TRUMP-Pricing(B,W,G,i)

$B^{'} = Sort(B \cup W)$;
if $State(i) == null$ **then**
  return $\tau_i = 0$;
τ_i=MAXNUM;
Set $d[k] = -1$ for $k \leftarrow 1$ **to** M;
Set $Avai(i) = \mathcal{M}$ for $i \leftarrow 1$ to N;
$B^{''} = Remove(B^{'}, i)$;
while $B^{''} \neq \phi$ **do**
  $q = Top(B^{''})$;
  $m = Assign(q)$;
  $Remove(B^{''}, q)$;
  case *1:* $q \in \mathcal{N}$ *and* $(i, q) \in \mathcal{E}$
    $j = q$, $k = 0$, $w = b^p_j$;
  case *2:* $q \in \mathcal{E}$ *and* $(i, q_1) \in \mathcal{E}$
    $j = q_1$, $k = q_2$, $w = w_q$;
  case *3:* $q \in \mathcal{E}$ *and* $(i, q_2) \in \mathcal{E}$
    $j = q_2$, $k = q_1$, $w = w_q$;
  if $d[m] < 0$ **then**
    $d[m] = w$;
  $\tau_i = \min(\tau_i, PriceTable(i, j, k, m))$;
if $|Avai(i)| \geq 1$ **then**
  $\tau_i = 0$;
return τ_i;

There are two cases for charging winner i. (1) $\tau_i = 0$; (2) $\tau_i = \min(\tau_i, PriceTable(\cdot))$. The former case indicates that i can always be allocated (penultimate line in Algorithm 3). This is because there are adequate channels for bidder i. Otherwise, i will compete channels with its neighbors. $PriceTable(\cdot)$ ensures that i can always seize one channel from its neighbors by bidding with

TABLE 2: $PriceTable(\cdot)$ for charging i in Fig. 2

Cases	x∈ TYPE-I	x∈ TYPE-II && $State(i) == P$	x∈ TYPE-II && $State(i) == S$
(1)(4)	$d[m]$	$d[m]$	$d[m] - b_j^p$
(2)(6)(7)(8)	$d[m] - b_j^s$	$d[m] - b_j^s$	$d[m] - b_j^p$
(3)(5)	impossible	impossible	impossible

a higher price than its neighbors. In details, if i wants to seize channel m from its neighbors, its bid or related edge's weight must exceed $d[m]$, which is the maximum weight/bid of i's neighbors attached on m in TRUMP-Pricing. Because bidder i needs only one channel, the final price is the minimal value of all computed τ_i, which means i cannot seize any channel by bidding lower than that. We list all combinatorial cases to seize a channel from its neighboring nodes or edges in Fig.2, and the corresponding prices are listed in Table 2. For instance, in case (4), if bidder i is of TYPE-II and wins primary access in TRUMP-Allocation, it has to bid higher than $d[m]$ to seize the channel m from neighbors; On the other hand, if i wins secondary access in TRUMP-Allocation, $b_i^s + b_j^p$ must be higher than $d[m]$ so as to seize the channel, *i.e.*, i must bid higher than $d[m] - b_j^p$ to share the channel with j.

It is a straightforward exercise to show that the complexity of procedures $Sort(\cdot)$, $Assign(\cdot)$, and $Remove(\cdot)$ are $O((N+L)\log(N+L))$, $O(L)$ and $O(N+L)$, respectively. In TRUMP-Allocation(B, W, G), $Sort(\cdot)$ is invoked once, while $Assign(\cdot)$ and $Remove(\cdot)$ are invoked N times. Its overall complexity is therefore $O((N+L)\log(N+L) + NL + N^2)$ time. In TRUMP-Pricing(B, W, G, i), the sorted bids from TRUMP-Allocation will be reused and hence its complexity mainly comes from invoking $Assign(\cdot)$ and $Remove(\cdot)$ in the while-loop. Because bidder i is excluded, the loop takes only $N-1$ iterations, leading to a complexity $O(NL+N^2)$ for each winner. Finally, since there will be at most N winners, the complexity of computing payment for all bidders is $O(N^3 + N^2 L)$. In summary, the complexity of TRUMP is $O((N+L)\log(N+L) + N^3 + N^2 L)$.

The analysis on truthfulness and individual rationality of our proposed mechanism can be found in the supplementary file. The major theoretical result is summarized below.

***Theorem** 2:* TRUMP is truthful and individually rational.

5 Extension of TRUMP

In this section, we consider more general cases, where a basic spectrum allocation can accommodate one primary user and at most r secondary users. To make the problem tractable, we assume that all the secondary users on a channel have equal access rights. This is because complicated multiple access protocols would be required to grant access at different priority levels to different secondary users on a channel. Under this consideration, we propose r-TRUMP based on TRUMP ($r \geq 1$), where the channel is said to be divided into r *secondary parts* for secondary sharing.

5.1 r-TRUMP Allocation Rules

We design the allocation rule based on the TRUMP-Allocation. We add an auxiliary number for each bidder i to indicate how many secondary bidders have been attached to primary bidder i. Moreover, we record which secondary bidders have been attached to primary bidder i. Therefore, we introduce an auxiliary array $aux[r+1]$, where $aux[0]$ records the total number of attached secondary bidders and remaining $aux[1]$ to $aux[r]$ record the index of attached bidders. The outline of r-TRUMP-Allocation algorithm is described as follows:

STEP 1: Execute TRUMP-Allocation algorithm, and create an array $aux[r+1]$ for each primary winner (the bidder that wins primary access right) to record the index of attached bidders.

STEP 2: For bidders of TYPE-II who have not been allocated in STEP 1, we sequentially handle them from the one with the highest bid (b_i^s) to the lowest one. For each bidder i to be allocated in STEP 2, we traverse its neighbors (including edges and nodes) in descending order and find the first primary winner j that can accommodate additional secondary bidders. Then we assign i secondary access on the channel.

Note that the following two conditions must be satisfied for bidder i to be attached on the primary winner j:

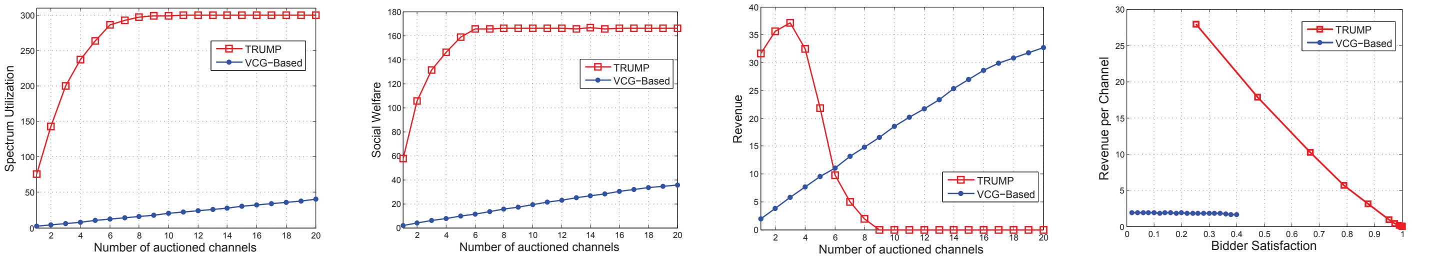
- (C1) other neighbors of i except j do not use the same channel with j (except those attached on j);
- (C2) $aux[0] < r$ for j.

The former condition ensures that the accommodation will not cause conflict to other bidders. The latter one ensures that there are at most r secondary bidders on each channel.

5.2 r-TRUMP Pricing Rules

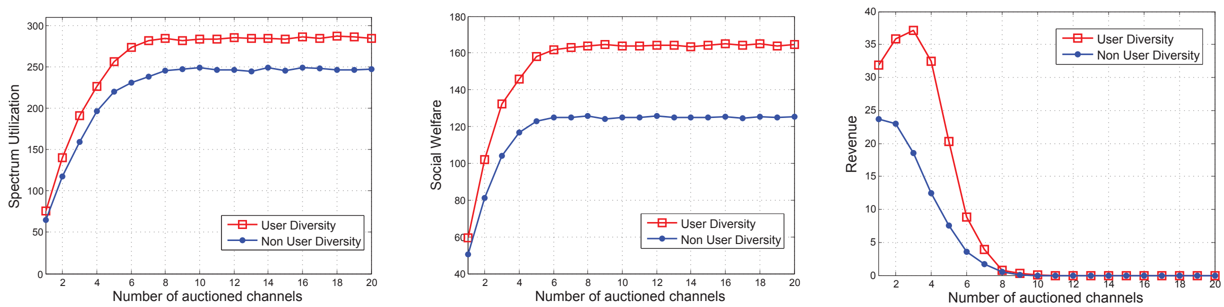
The basic idea of r-TRUMP-Pricing is similar to TRUMP-Pricing algorithm, *i.e.*, first taking i and its related edges out of the sorted list B', and then performing r-TRUMP-Allocation again to find the charged price. When assigning a channel to i's neighbors, the minimal bid for seizing one of its neighbor's channel is the charged price. The only difference is that r-TRUMP-Pricing would charge a lower price for bidders of TYPE-II since the competition reduces when more secondary bidders could be allocated.

Let $\hat{\tau}_i$ be the price charged to i in r-TRUMP-Pricing. After running TRUMP-Pricing on i, $\hat{\tau}_i$ is initialized as $\hat{\tau}_i = \tau_i$. If i wins a primary right (*i.e.*, i has been allocated in STEP 1 of r-TRUMP-Allocation), then r-TRUMP-Pricing completes for bidder i. Otherwise, we set $\hat{\tau}_i$ to the minimal bid such that i can win a secondary right in STEP 2 of r-TRUMP-Allocation.



(a) Spectrum Utilization (b) Social Welfare (c) Revenue (d) Revenue vs. Bidder Satisfaction

Fig. 3: Performance comparison between TRUMP and an VCG-Based auction [9] which does not exploit the spatial reuse of spectrum.



(a) Spectrum Utilization (b) Social Welfare (c) Revenue

Fig. 4: Performance improvement of TRUMP by exploiting user diversity.

For the latter case, we need to check whether bidder i can be accommodated by one of its primary neighbors, *e.g.*, j, after running r-TRUMP-Allocation on bidders except i (*i.e.*, $\mathcal{N}\backslash\{i\}$). It is conducted by traversing all its neighbors in a descending order on their bids. If there exists any neighbor k of i (not attached on j) that disqualifies condition (C1), we set

$$\hat{\tau}_i = \min(\hat{\tau}_i, b_k^s) \tag{11}$$

to beat k such that i will seize the channel and (C1) be guaranteed if bidding at $\hat{\tau}_i$. Otherwise, we update $\hat{\tau}_i$ as follows such that condition (C2) will hold as well.

$$\hat{\tau}_i = \begin{cases} 0, & aux[0] < r \\ \min_{k=1}^{r}(\hat{\tau}_i, b_{aux[k]}^s), & \text{otherwise}. \end{cases} \tag{12}$$

The setting $\hat{\tau}_i = 0$ indicates that i will be always allocated since j has accomodated less than r secondary bidders, *i.e.*, $aux[0] < r$. Otherwise, i has to bid the lowest bids, *i.e.*, $\min_{k=1}^{r}(b_{aux[k]}^s)$, among r secondary bidders attached to j. After checking all neigbors of i, the final updated $\hat{\tau}_i$ is the price charged to i.

Similarly, we show the properties of our extended auction with a proof given in the supplementary file.

***Theorem** 3:* The r-TRUMP is truthful and individually rational.

6 Performance Evaluation

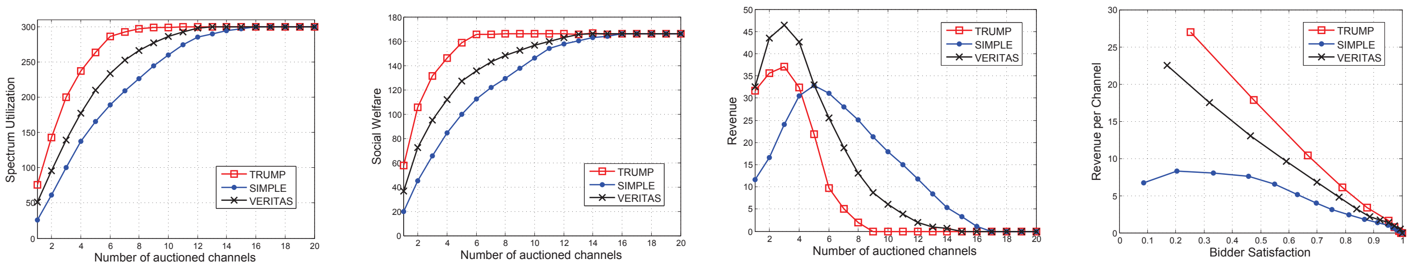
In this section, we evaluate the performance of TRUMP via simulations. First, we examine the impact of spacial reuse and user diversity to the performance. Then, we extensively evaluate the performance of TRUMP and r-TRUMP. Finally, we investigate the fairness issue of our proposal.

6.1 Simulation Methodology

We assume a single auctioneer that conducts an auction in a relatively small geographic area. Bidders are randomly deployed in a square 1.0×1.0 area. By default, 300 bidders are deployed and the number of bidders of each type is random. To generate the interference graph, we set the interference range as 0.1, *i.e.*, if the distance between any two bidders is less than 0.1, they will interfere with each other when using the same channel simultaneously.

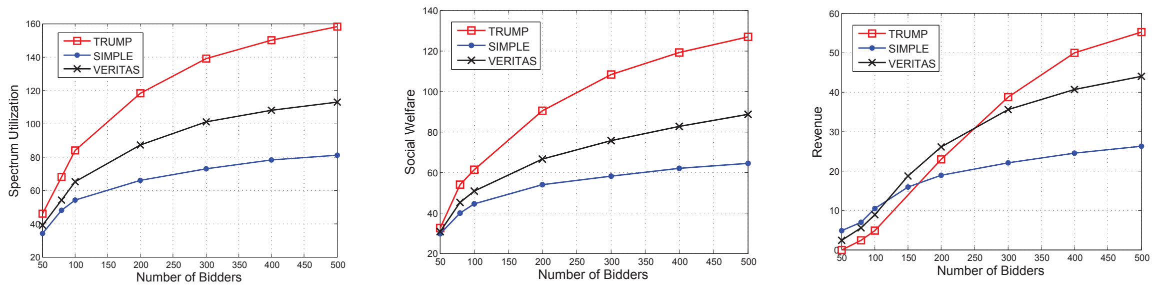
We set up an auction of total channels varying from 2 to 20 channels. Each bidder independently submits a bid tuple $b_i = \langle b_i^p, b_i^s \rangle$ to the auctioneer. For bidder of TYPE-I, b_i^p is uniformly distributed over (0,1], and the b_i^s is set to -9999. For bidders of TYPE-II, both b_i^p and b_i^s is uniformly distributed over (0,1] and $b_i^p \geq b_i^s$ is ensured. To eliminate the impact of randomness, the results are averaged over 10 random seeds. We use the following performance metrics to evaluate the performance of TRUMP and r-TRUMP:

- **Social Welfare**: The sum of all winners' bids.
- **Revenue**: The sum of total charged payments from winners.



(a) Spectrum Utilization (b) Social Welfare (c) Revenue (d) Revenue vs. Bidder Satisfaction

Fig. 5: Performance comparison between TRUMP, SIMPLE and VERITAS by auctioning 1-20 channels.



(a) Spectrum Utilization (b) Social Welfare (c) Revenue

Fig. 6: Performance comparison between TRUMP, SIMPLE and VERITAS with different bidders.

- **Spectrum Utilization**: The sum of allocated channels over winners, *i.e.*, the number of winners.
- **Bidder Satisfaction**: The percentage of winners.

6.2 Spatial Reuse vs Non-Spatial Reuse

We compare TRUMP with a VCG-based auction [9] which consists of an allocation algorithm and a VCG pricing scheme. The allocation algorithm is based on finding maximum weighted matching [18]. The results of both auctions are shown in Fig. 3(a)-(d).

From Fig. 3(a) and Fig. 3(b), we observe that TRUMP significantly outperforms the VCG-based auction in spectrum utilization and social welfare. This is attributed to the fact that TRUMP exploits the spatial reusability with consideration of the interference constraints while the VCG-based auction does not. In VCG-based auction a channel can be allocated to at most two bidders and thus the spectrum utilization and revenue of the VCG-based auction grow linearly with the growing number of channels.

In Fig. 3(c), we observe that although the maximal revenues of both auctions are similar, TRUMP achieves the maximum using 3 channels while the simple design requires 20 channels. Furthermore, the revenue decreases as long as the number of channels to be auctioned exceeds a certain value. The decreasing of revenue is due to the shrink of the number of losing bidders. That is, we can improve the total revenue by controlling competition among bidders, for example, to control the number of auctioned channels or to attract more bidders to participate in the auction.

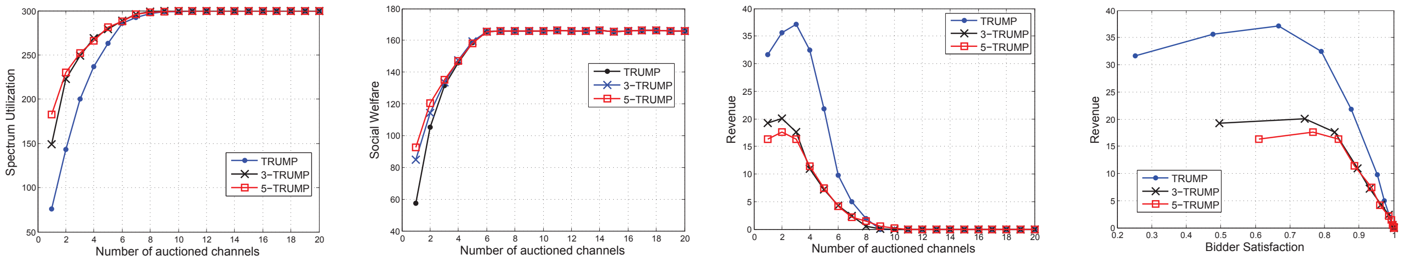
For a fair comparison, we plot the revenue per channel as a function of the bidder satisfactory rate by varying the number of channels auctioned (see Fig. 3(d)). Again, TRUMP significantly outperforms the VCG-based design.

6.3 User Diversity vs Non-User Diversity

Now we explore the benefits brought by supporting QoS-aware bidding. We first run TRUMP on a random mix of bidders with different types. Then, we perform TRUMP again with these bidders by setting the secondary bid of each bidder equal to its primary bid. If a TYPE-I bidder is allocated with secondary right, the utilization is set to zero and the bidder is charged zero. We plot the results in Fig. 4(a)-(c). We observe that TRUMP significantly improves spectrum utilization and social welfare while addressing user diversity by up to 25% and 35%, respectively.

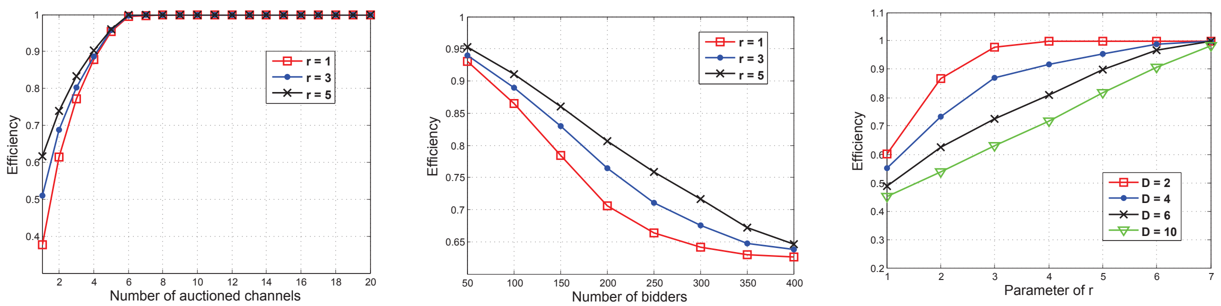
6.4 Performance of TRUMP

Now we evaluate the performance of TRUMP by comparing to SIMPLE and VERITAS [6] that both consider spatial reuse. The former is an extension of the traditional scheme [9] by dividing the whole region into boxes with length of the maximal interference radius such that bidders in each box conflict with each other and the VCG-based auction can be applied directly in each box.



(a) Spectrum Utilization (b) Social Welfare (c) Revenue (d) Revenue vs. Bidder Satisfaction

Fig. 7: Performance of r-TRUMP under various settings of parameter r.



(a) (b) (c)

Fig. 8: Examine the effect of parameter r.

The latter is the most famous SSA which also takes interference into account. In order to adapt VERITAS to our scenario, we run VERITAS on the same bids where only primary bids are effective. The performance comparison is conducted under various numbers of auctioned channels and bidders. These experiments allow us to understand these mechanisms under different levels of resource contention.

The results under various number of channels are shown in Fig. 5, where TRUMP significantly improves the performance compared to VERITAS and SIMPLE. This is due to the fact that TRUMP allows more bidders to be allocated compared to VERITAS. SIMPLE ensures the truthfulness while suffering from significant degradation in social welfare and spectrum utilization. To provide a more fair comparison on the generated revenue, we further plot the revenue per channel as a function of the bidder satisfactory rate by varying the number of channels auctioned. As shown in Fig. 5(d), we observe that TRUMP outperforms both VERITAS and SIMPLE once again.

From the results under various number of bidders as shown in Fig. 6(a)-(b), we see that TRUMP again significantly improves the social welfare and spectrum utilization. In Fig. 6(c), TRUMP generates lower revenue than VERITAS and SIMPLE when the number of bidders is small. This is because more bidders are charged zero in TRUMP when the contention level is low. However, when the number of bidders is large, *i.e.*, the contention of channels is intensive, TRUMP generates higher revenue. This is because more bidders are accommodated and thus charged in TRUMP as confirmed in Fig .5(d). Moreover, the performance improvement generally increases as more bidders are involved.

6.5 Perforamnce of r-TRUMP

We evaluate r-TRUMP by tuning the parameter r using three different values: TRUMP, 3-TRUMP and 5-TRUMP. The results are shown in Fig. 7. We observe that the spectrum utilization and social welfare increase with the increment of r. The results confirm that more bidders can be allocated with secondary access. Similar as TRUMP, the revenue turns to decrease once the number of channels to be auctioned exceeds one certain value. Furthermore, we find that the performance gain is not obvious when r is larger than 3.

We then examine the *efficiency* of r-TRUMP, which is defined as the ratio of the sum of social welfare of winning participants to the total social welfare. It reflects the portion of bidder demands that are satisfied, weighted by bid vectors. The results under various values of r are plotted in Fig. 8(a). We observe that the efficiency increases with the increment of r. The results confirm that more bidders can be allocated with secondary access when r grows. Furthermore, we find that the performance gap is not obvious when r increases. This is because most bidders will be allocated when r exceeds a certain threshold.

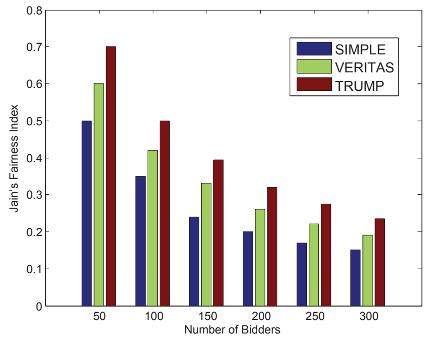


Fig. 9: Jain's fairness index with different numbers of bidders enrolled.

We then vary the number of bidders in the simulations. The results plotted in Fig. 8(b) show that the improved efficiency by exploiting larger r will be maximized at a moderate number of bidders, *e.g.*, N =200-250. Too few or too many bidders lead to similar efficiency under various values of r.

Based on the above observations, we conclude that r is relevant to the average degree $D = \sum_{i\in\mathcal{N}} |N(i)|/N$ of conflict graph. The result in Fig. 8(c) confirms our deduction. In summary, r is a tunable parameter which is highly relevant to the underlying infrastructure. It can be adjusted according to the conflict conditions.

6.6 Fairness

In this section, we quantify the fairness of our method by using Jain's fairness index [22]. The Jain's fairness index is computed as $\frac{1}{N}\sum_{i=1}^{N}\frac{X_i}{X_f}$, where X_i denotes the allocation of bidder i and $X_f = \frac{\sum_{i=1}^{N} X_i^2}{\sum_{i=1}^{N} X_i}$ is the *fair allocation mark*. Thus each bidder compares his allocation X_i with the amount X_f, and perceives the algorithm as fair or unfair depending upon whether his allocation X_i is more or less than X_f. The overall fairness is the average of *perceived fairness* of total N bidders.

The results are plotted in Fig. 9. From the results, we can observe that TRUMP performs better than SIMPLE and VERITAS. This is because the Jain's fairness index computed as bidder satisfaction in our settings for each method, where X_i is 1 if bidder i wins and is 0 if it loses. Therefore the result is contributed by the fact that more bidders can be allocated in TRUMP, either be primary or be secondary.

7 Conclusion

In this paper, we propose TRUMP, a QoS-aware truthful spectrum auction framework for single-step access allocation in CRNs. TRUMP allows bidders to independently express bids for their valuations on spectrum in terms of QoS demands. Furthermore, it exploits the spatial reuse by addressing interference to improve spectrum utilization. We extend TRUMP to r-TRUMP so as to serve more secondary users according to network usage. Through theoretical analysis, we prove that both TRUMP and r-TRUMP are truthful and computational-efficient. We also validate the performance of TRUMP and r-TRUMP using extensive simulations.

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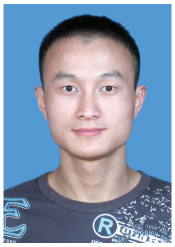

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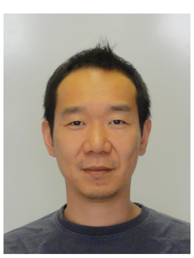

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