A Test Oracle for Reinforcement Learning Software based on Lyapunov Stability Control Theory


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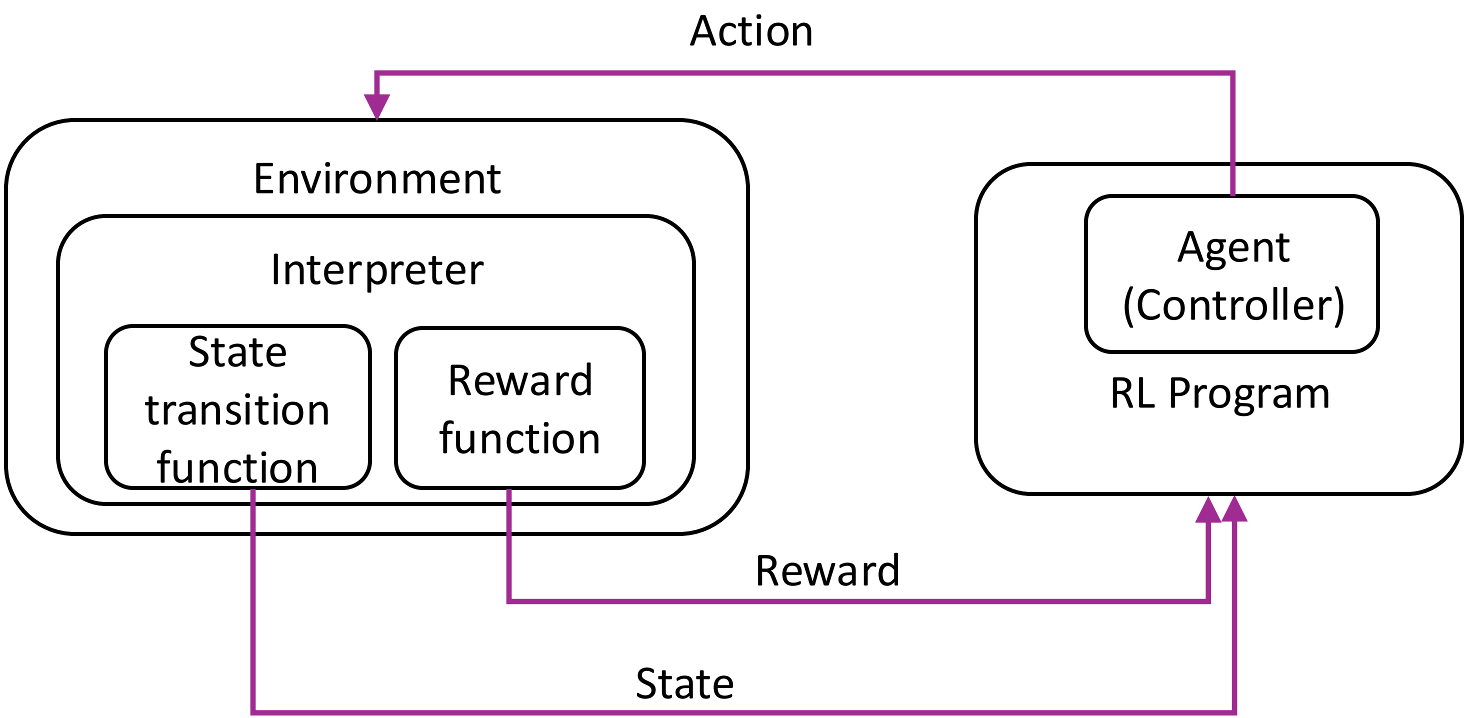
Reinforcement Learning (RL) becomes increasingly important due to its ability to train controllers for complex systems/robots.



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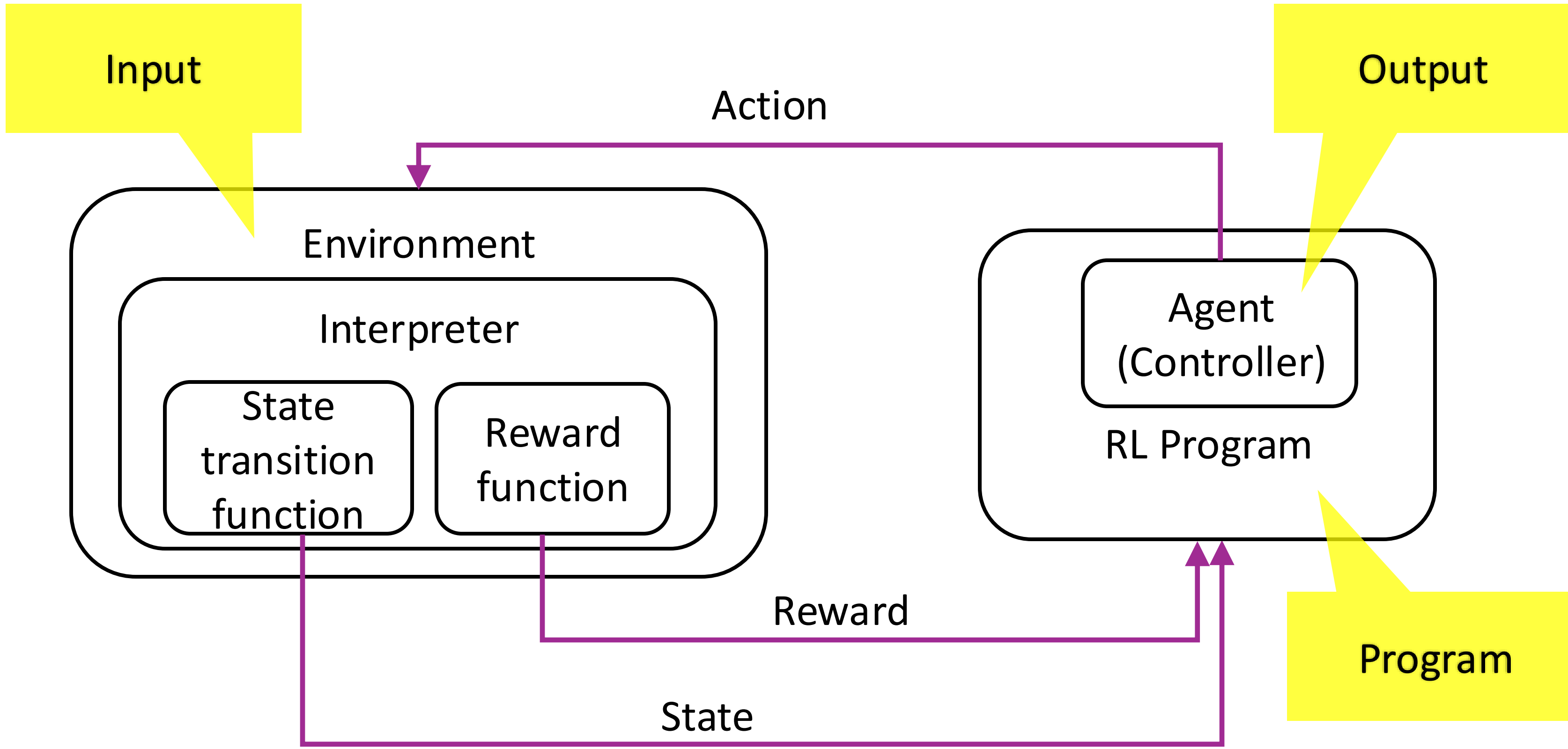

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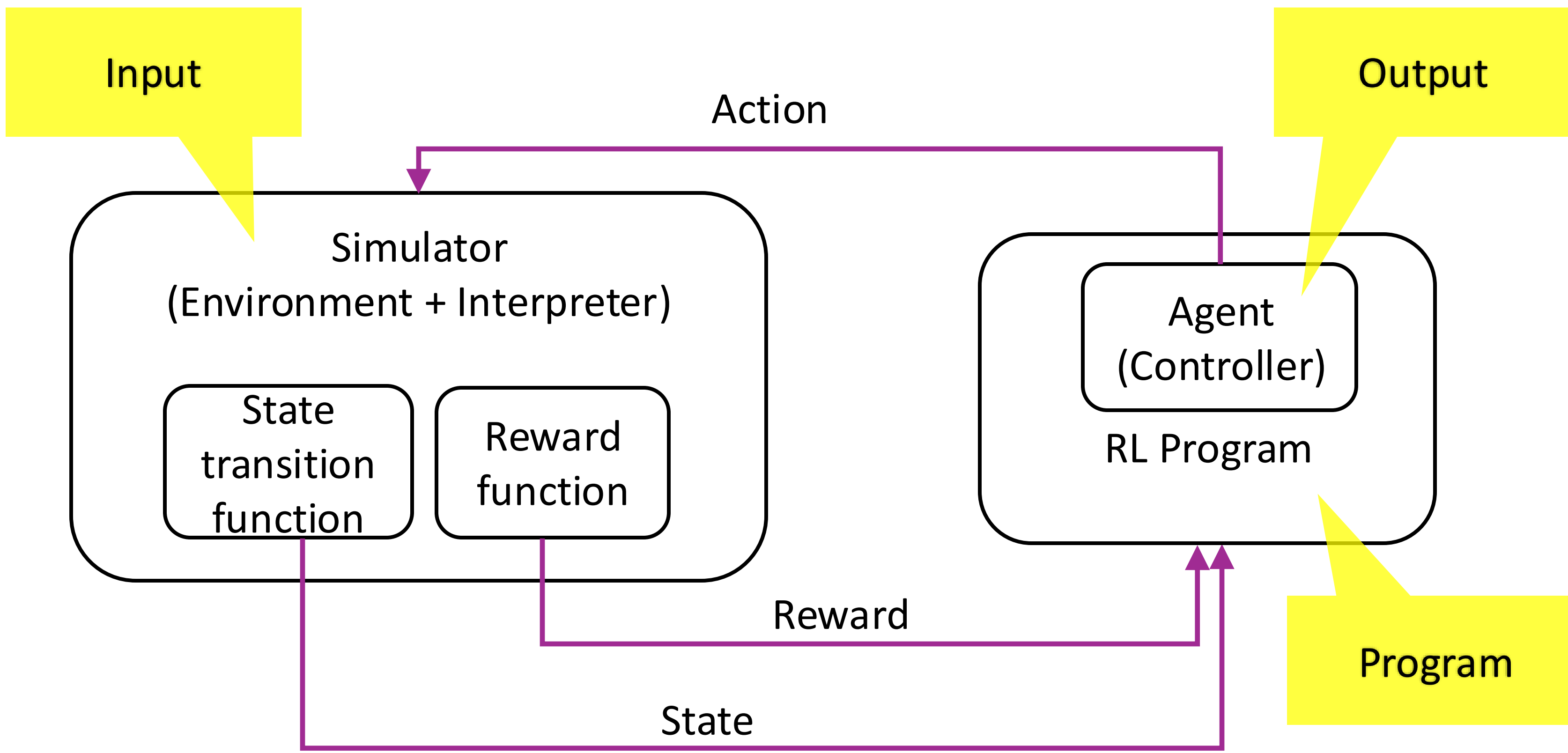
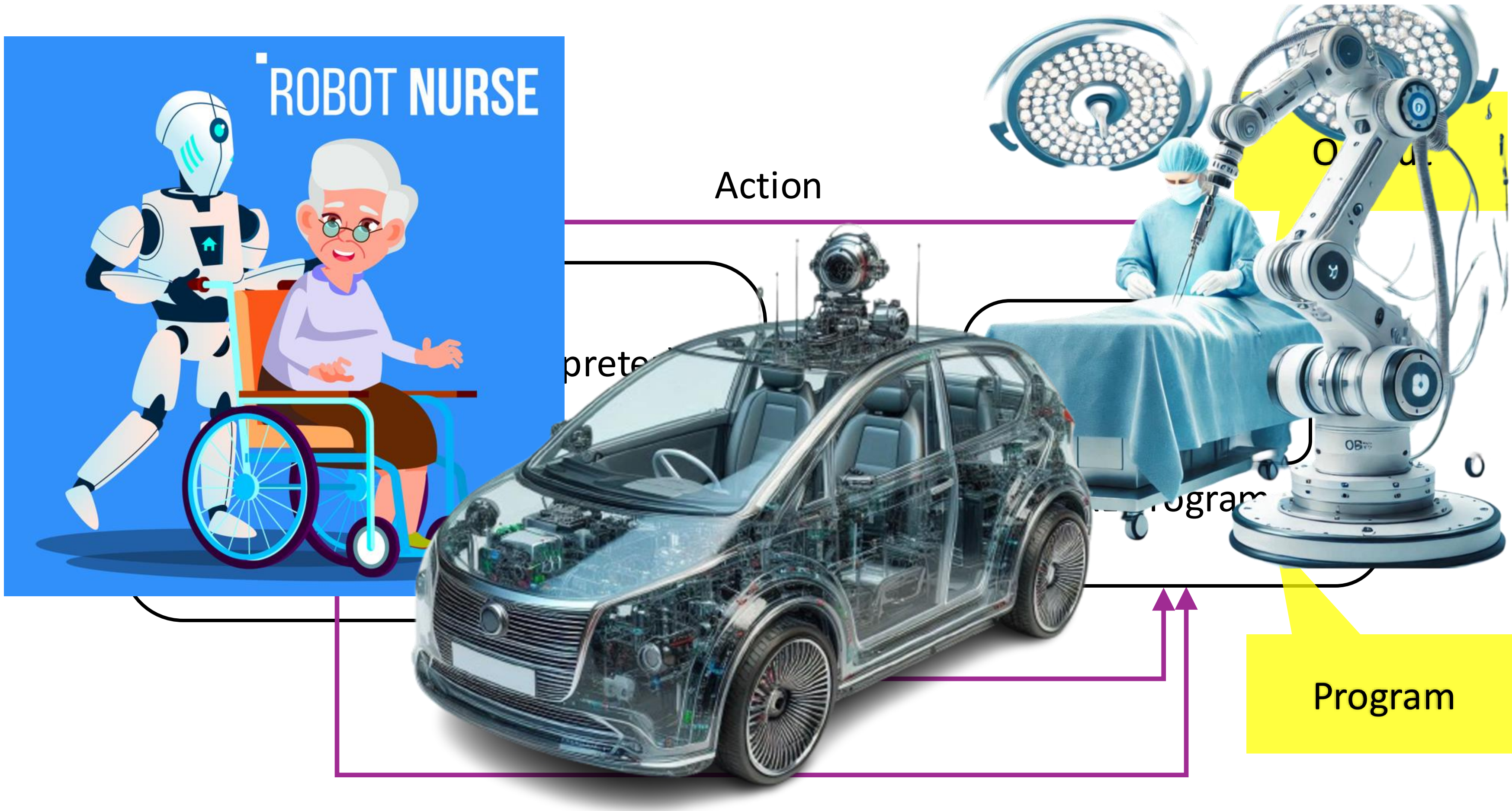
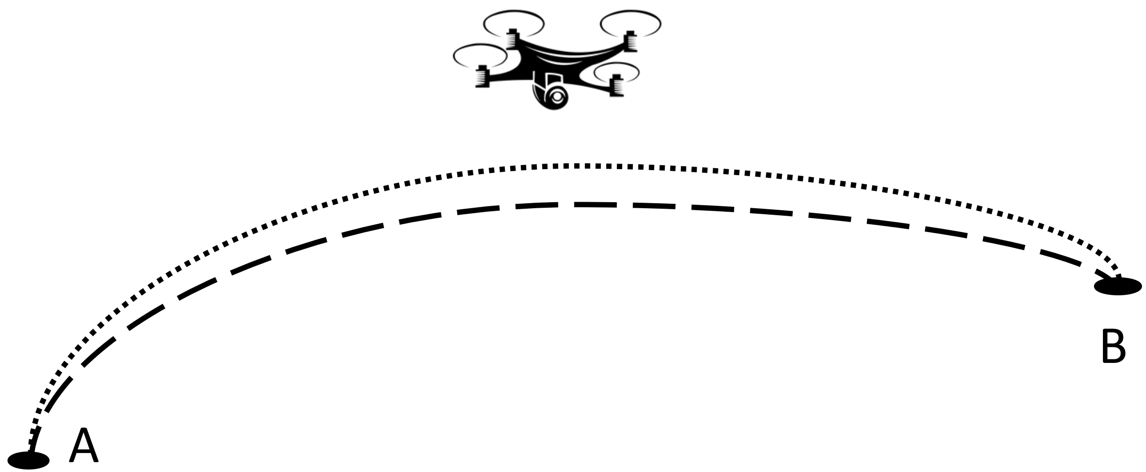


As RL generates controllers for critical control/robotic applications, ensuring RL program correctness is important.



Oracle Problem for RL: how to judge if an RL program is buggy (in other words, its generated controller is wrong)?

The correct controller to a system/robot is not unique.



Less overshoot? Or less time cost? Or less jitter?

Solution Heuristics

Create many non-trivial controller design problems with well-known analytical/numerical solution and solution features.

If the RL generated controllers most often agree with the well-known analytical/numerical solution features, then we label the RL program correct, and otherwise buggy.

Lyapunov stable controller design theory

Given: $\dot{X} = AX + BU$, where $X \in \mathbb{R}^n$, $U \in \mathbb{R}^m$

State of the Plant

Control

Lyapunov stable controller design theory

Given: $\dot{X} = AX + BU$, where $X \in \mathbb{R}^n$, $U \in \mathbb{R}^m$

Demand: $U = -KX$

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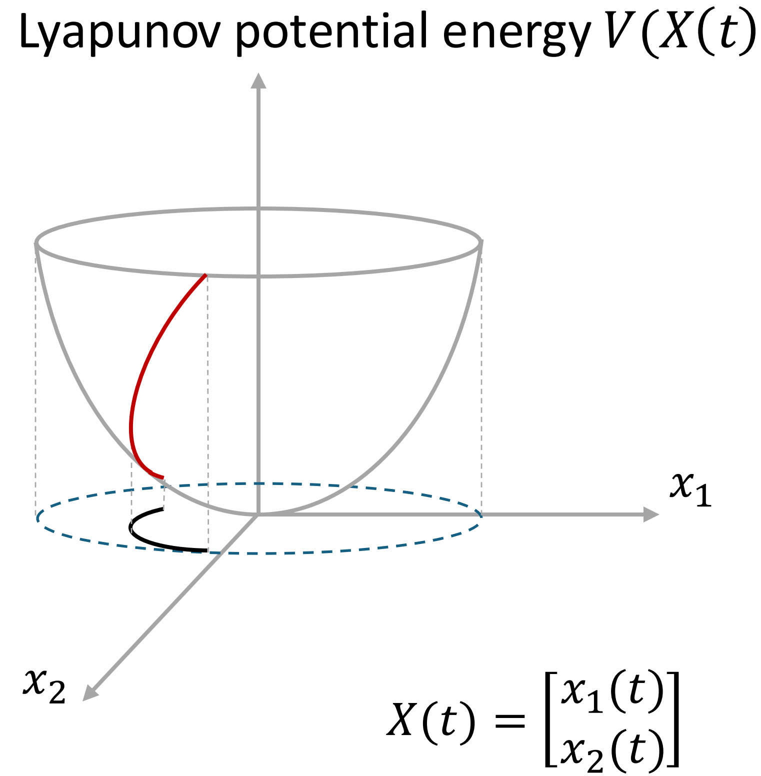


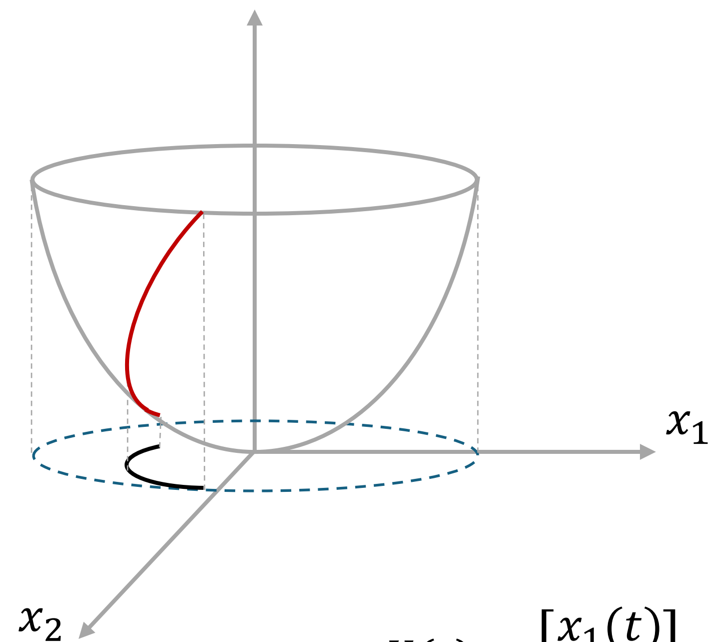
Lyapunov stable controller design theory

Given: $\dot{X} = AX + BU$

Demand: $U = -KX$

Well-known numerical solution exists.

Lyapunov potential energy $V(X(t))$



$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

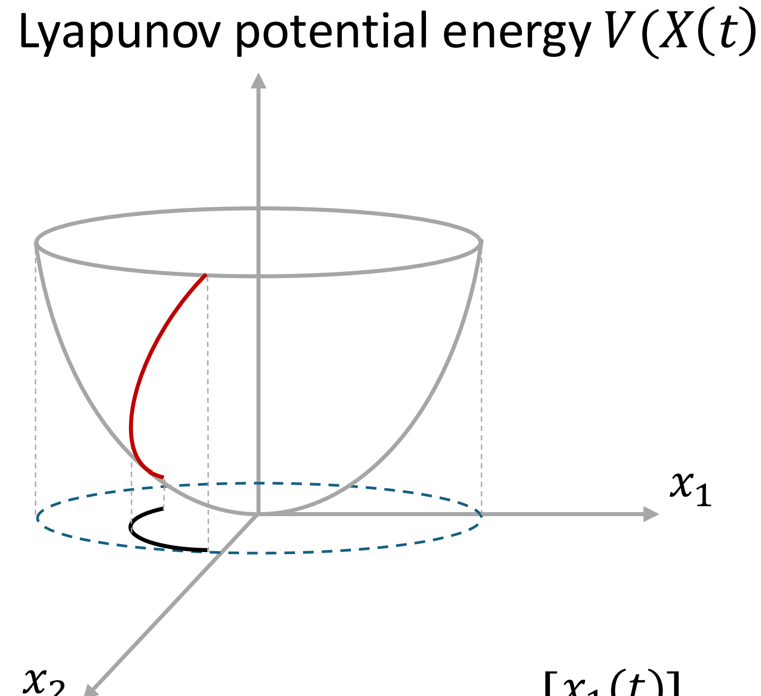
Lyapunov stable controller design theory

Given: $\dot{X} = AX + BU$

Demand: $U = -KX$

Well-known numerical solution exists.

Solution feature: the so-designed controller U always decreases the plant's Lyapunov potential energy $V(X(t))$ over time.



$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

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Given: $\dot{X} = AX + BU$

Prepare: $U = -KX$

Create many non-trivial Lyapunov stable controller design problems.

Prepare the ground truth Lyapunov stable controller U and Lyapunov potential energy $V(X(t))$ as per the conventional numerical solution.

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During RL Training: Use the ground truth Lyapunov potential energy $V(X(t))$ decreasing feature to guide the reward generation.

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The RL is considered bugless iff over θ percentage of its generated controllers are complying.

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Lyapunov Potential Energy Abnormality Oracle: LPEA(ϑ, θ).

Evaluation: workflow

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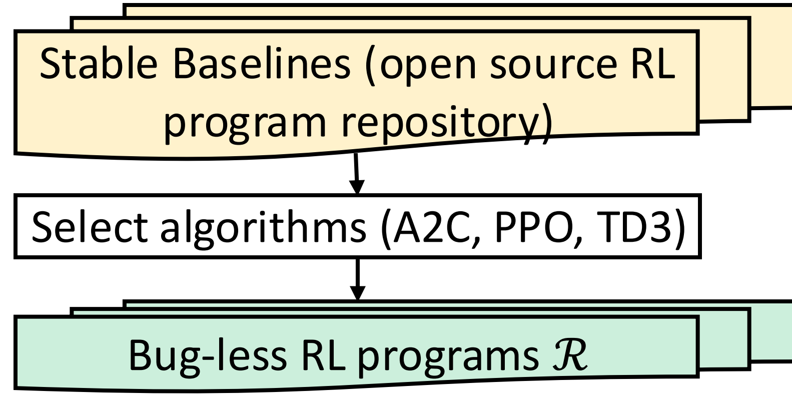
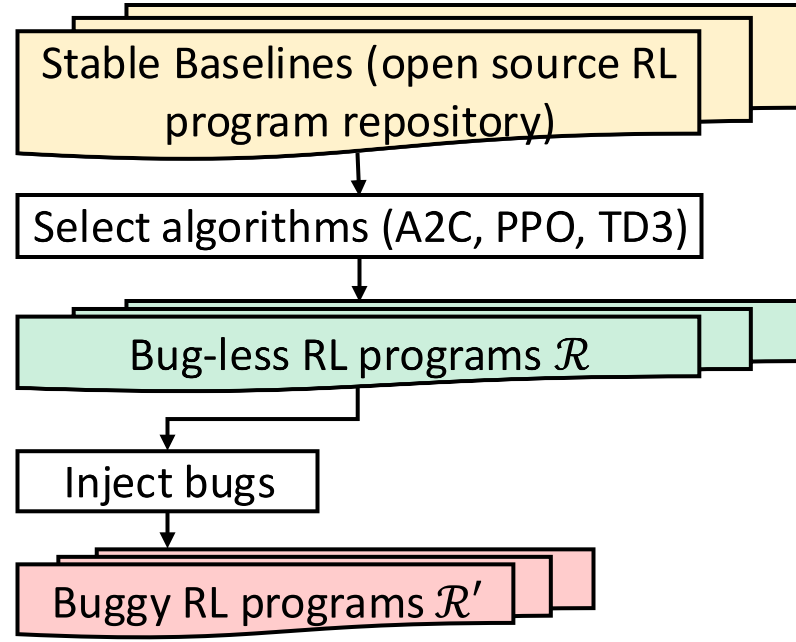
Stable Baselines (open source RL program repository)

Evaluation: workflow



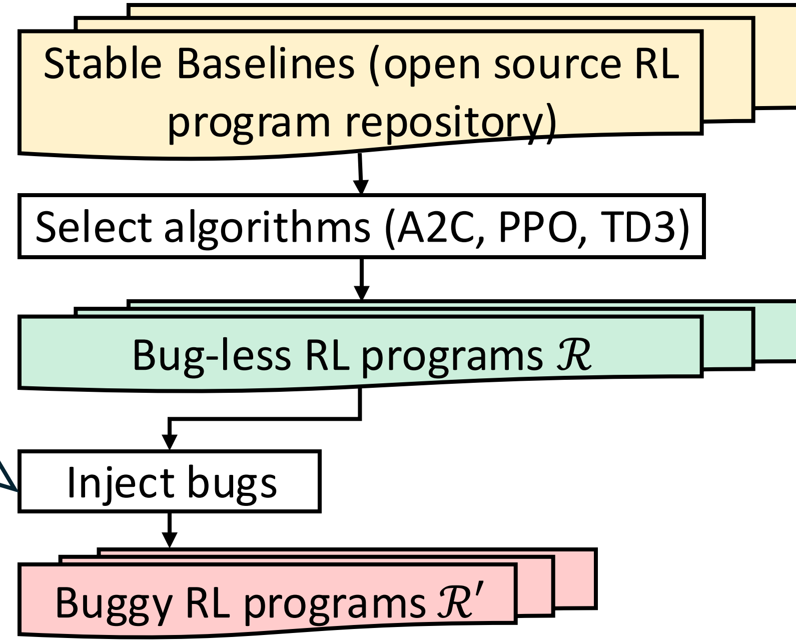
Evaluation: workflow



Evaluation: workflow

TABLE I: RL Software Bug Types from Survey [19]

Level-1 Type	Level-2 Type
1. RL Specific	1.1. Exploring the environment bugs
	1.2. Updating network bugs
2. NN Specific	2.1. Model bugs
	2.2. Training bugs
	2.3. Tensor and inputs bugs



Evaluation: workflow

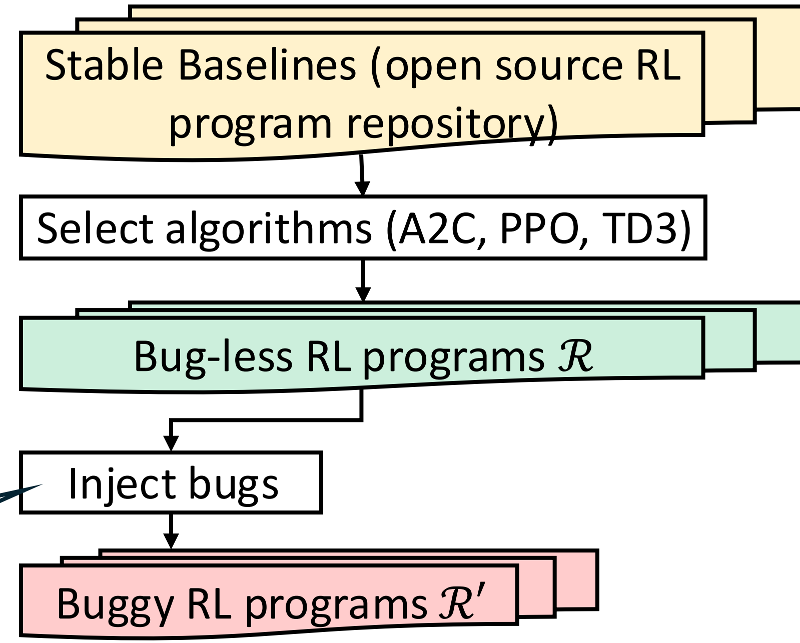
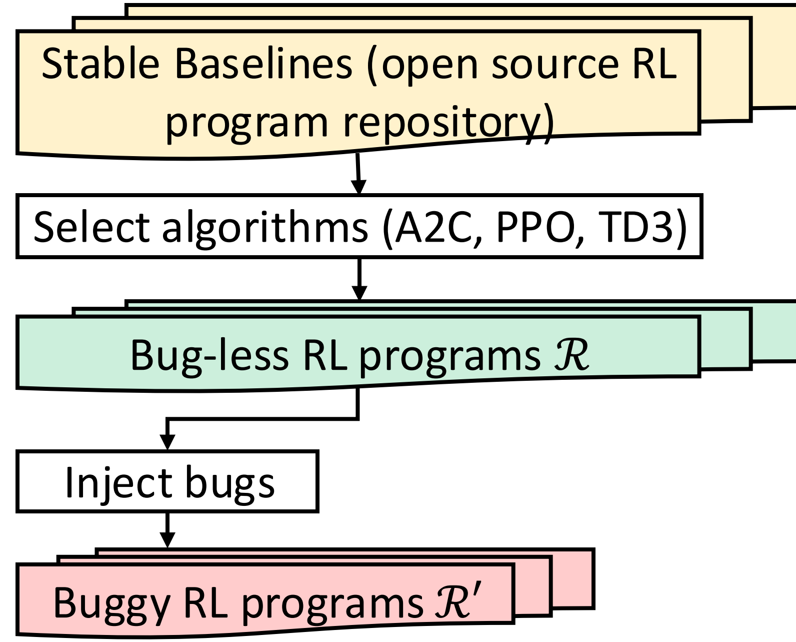


TABLE III: Real-World Bugs

Bug ID	Description
RW1	SB3 GitHub commit hash: `d5986da` File: `stable_baselines3/common/on_policy_algorithm.py` Line 167: “`dones`” should be “`_last_dones`”; Line 178: should insert one more line “`self._last_dones = dones`” before this line.
RW2	SB GitHub commit hash: `689afd1` File: `stable_baselines/a2c/a2c.py` Line 325: “`np.int32`” should be “`self.env.action_space.dtype`”.

Evaluation: workflow



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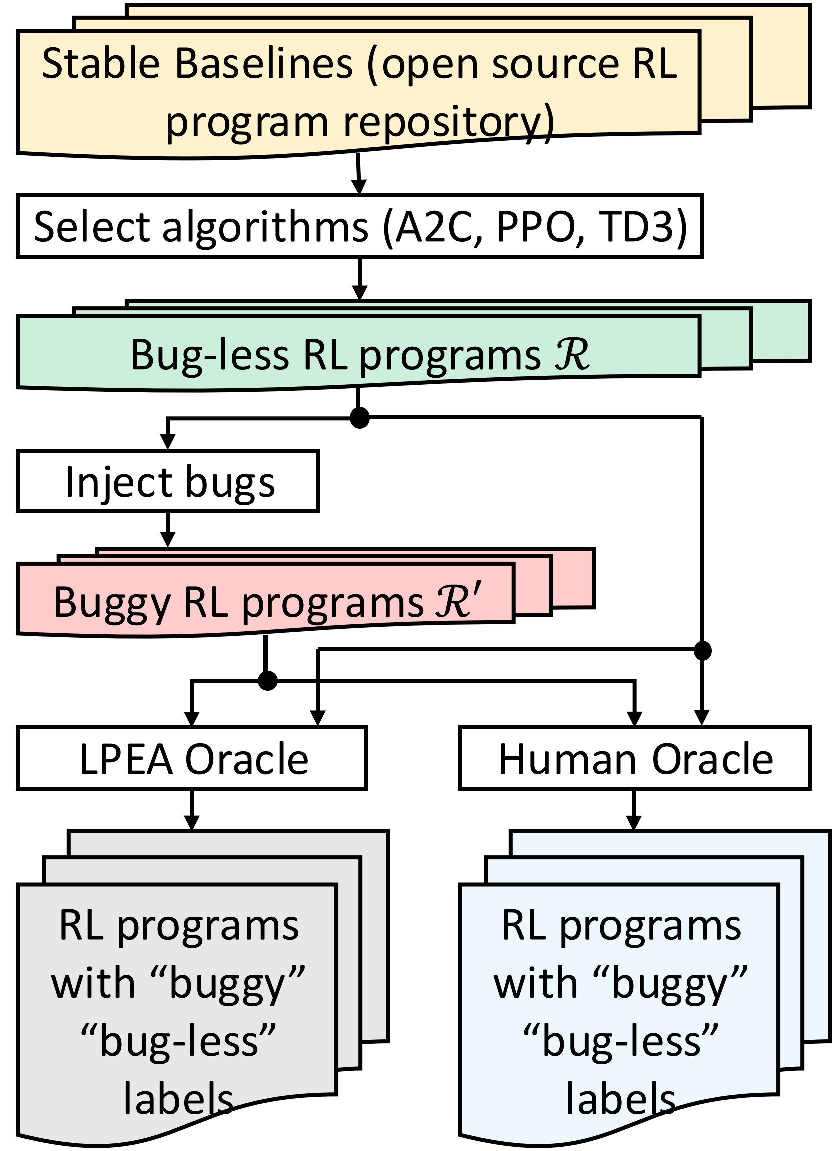
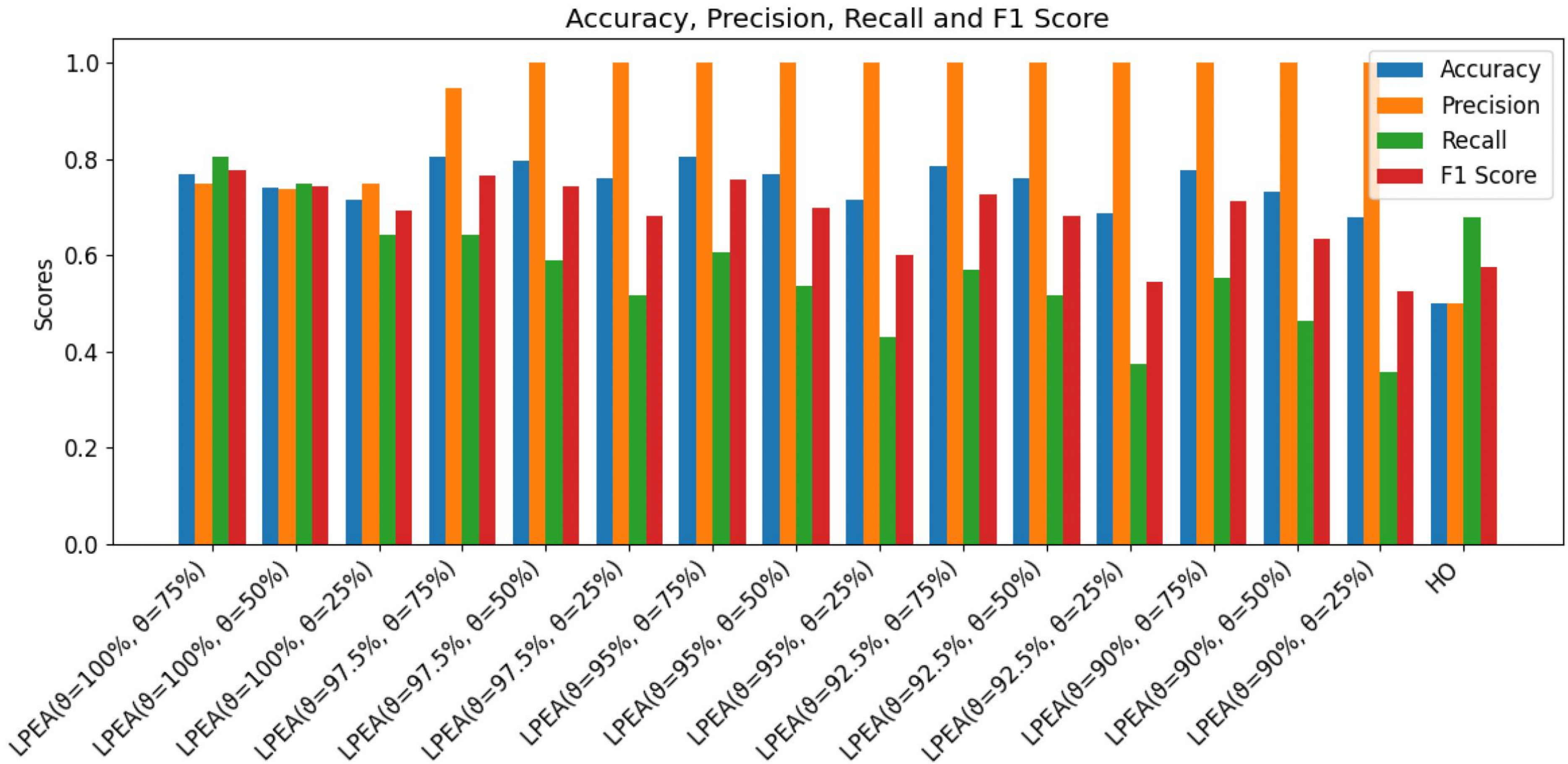


Evaluation: raw results

Oracle	A2C				PPO				TD3				Overall (A2C, PPO, and TD3)			
	TP	FP	TN	FN	TP	FP	TN	FN	TP	FP	TN	FN	TP	FP	TN	FN
LPEA($\vartheta = 100\%, \theta = 75\%$)	15	0	19	4	15	0	22	7	15	15	0	0	45	15	41	11
LPEA($\vartheta = 100\%, \theta = 50\%$)	15	0	19	4	12	0	22	10	15	15	0	0	42	15	41	14
LPEA($\vartheta = 100\%, \theta = 25\%$)	13	0	19	6	9	0	22	13	14	12	3	1	36	12	44	20
LPEA($\vartheta = 97.5\%, \theta = 75\%$)	15	0	19	4	9	0	22	13	12	2	13	3	36	2	54	20
LPEA($\vartheta = 97.5\%, \theta = 50\%$)	15	0	19	4	8	0	22	14	10	0	15	5	33	0	56	23
LPEA($\vartheta = 97.5\%, \theta = 25\%$)	13	0	19	6	6	0	22	16	10	0	15	5	29	0	56	27
LPEA($\vartheta = 95\%, \theta = 75\%$)	15	0	19	4	8	0	22	14	11	0	15	4	34	0	56	22
LPEA($\vartheta = 95\%, \theta = 50\%$)	13	0	19	6	7	0	22	15	10	0	15	5	30	0	56	26
LPEA($\vartheta = 95\%, \theta = 25\%$)	11	0	19	8	5	0	22	17	8	0	15	7	24	0	56	32
LPEA($\vartheta = 92.5\%, \theta = 75\%$)	13	0	19	6	8	0	22	14	11	0	15	4	32	0	56	24
LPEA($\vartheta = 92.5\%, \theta = 50\%$)	13	0	19	6	6	0	22	16	10	0	15	5	29	0	56	27
LPEA($\vartheta = 92.5\%, \theta = 25\%$)	9	0	19	10	5	0	22	17	7	0	15	8	21	0	56	35
LPEA($\vartheta = 90\%, \theta = 75\%$)	13	0	19	6	8	0	22	14	10	0	15	5	31	0	56	25
LPEA($\vartheta = 90\%, \theta = 50\%$)	13	0	19	6	5	0	22	17	8	0	15	7	26	0	56	30
LPEA($\vartheta = 90\%, \theta = 25\%$)	8	0	19	11	5	0	22	17	7	0	15	8	20	0	56	36
Human Oracle	12	15	4	7	15	15	7	7	11	8	7	4	38	38	18	18

Evaluation: results



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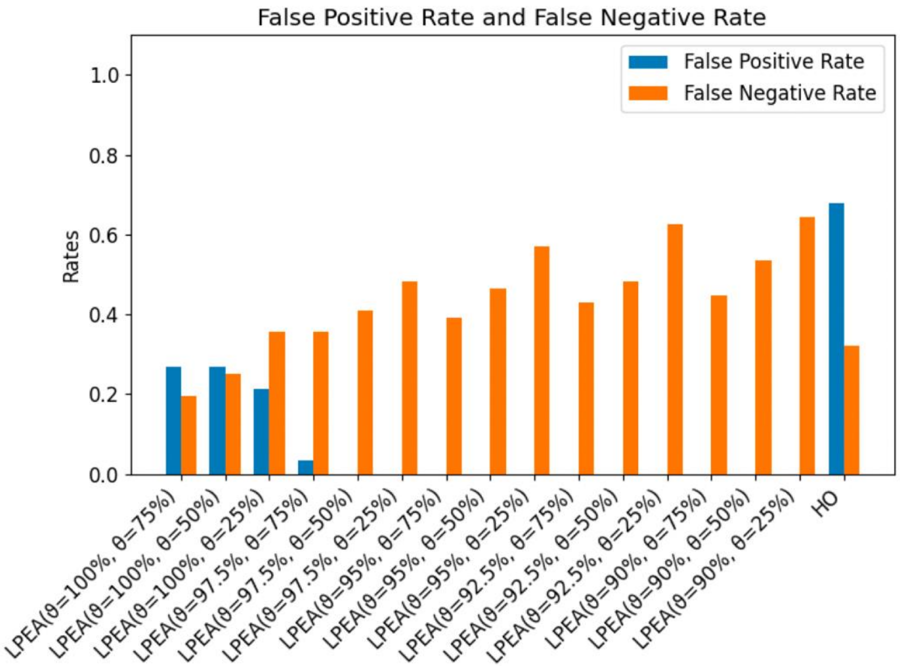
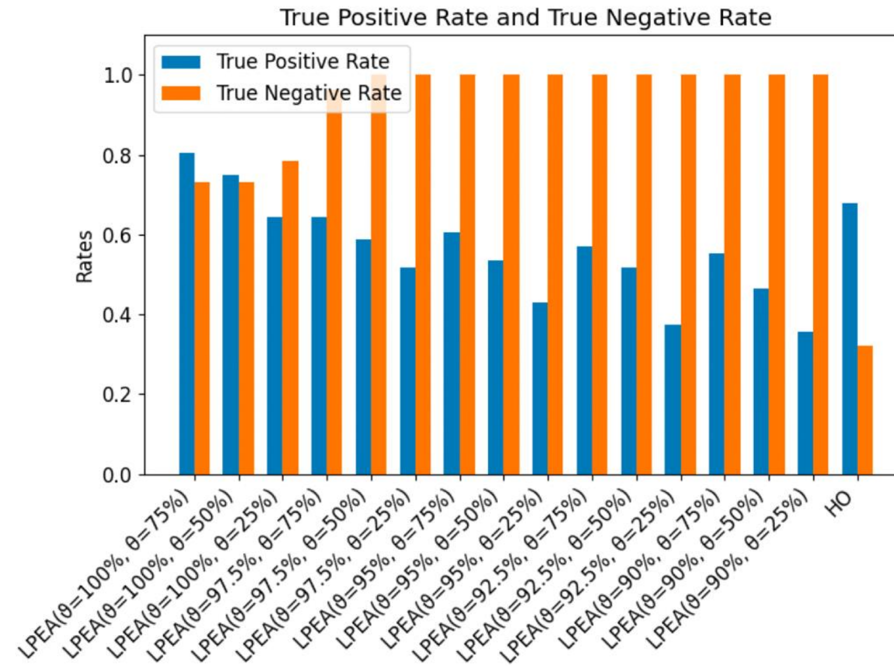
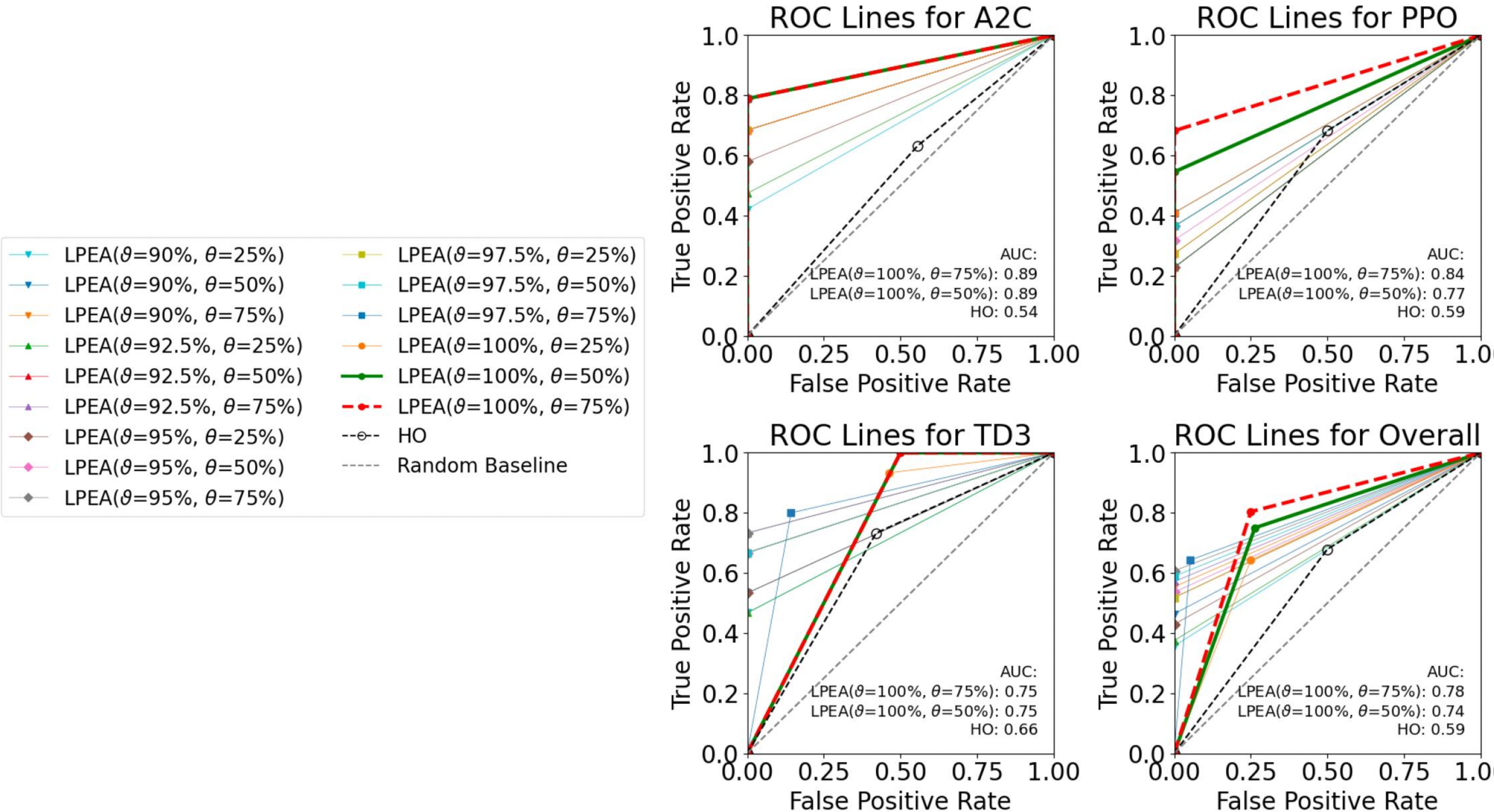


(b) False Positive Rate and False Negative Rate; lower values are better. HO: Human Oracle.



(c) True Positive Rate and True Negative Rate; higher values are better. HO: Human Oracle

⚖ Evaluation: results



Related Work

Metamorphic testing: LPEA oracle is a metamorphic testing oracle for RL, which is neither supervised learning or unsupervised learning (Xie et al. [26][27]).

Pang et al. [28] and Tappler et al. [29]: assume an obvious faulty state exists to define the control crashed.

Padgham et al. [30] needs white-box access to the design and implementation of the agents.

Nikanjam et al. [19] (DRLinter): static analysis upon RL program.

Varshosaz et al. [31]: needs white-box formal model.

LPEA oracle can be integrated into frameworks of property based testing, such as QuickCheck [32].

We can also use the formal language proposed by Jothimurugan et al. [33] to specify RL algorithms.

Shen et al. [34] and Hu et al. [35]: focus on evaluating test set quality; needs white-box controller.

Wan et al. [37]: focuses on improving the quality of test sets.

Lacoste et al. [38]: focuses on environmental costs of machine learning

Conclusion

LPEA oracles outperform the human oracle in most of the metrics.

Particularly, LPEA($\vartheta = 100\%, \theta = 75\%$) outperforms the human oracle by 53.6% in accuracy, 50% in precision, 18.4% in recall, 34.8% in F1 score, 18.4% in true positive rate, 127.8% in true negative rate, 60.5% in false positive rate, 38.9% in false negative rate, and 31.7% in ROC curve AUC.

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Thank You!