Supplementary Material to "Point Matching in the Presence of Outliers in Both Point Sets: A Concave Optimization Approach"

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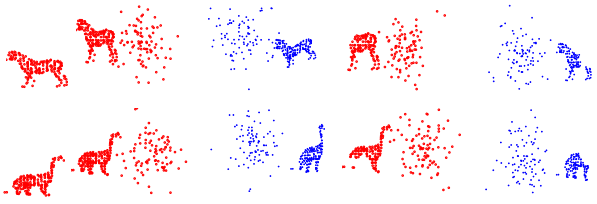
1 Experiments on synthetic 3D data set

For a 3D transformation consisting of scaling and translation, $T(\mathbf{x}_i|\boldsymbol{\theta}) = \begin{bmatrix}\theta_1 x_i^1 + \theta_4, & \theta_2 x_i^2 + \theta_5, & \theta_3 x_i^3 + \theta_6\end{bmatrix}^T$ with $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_6]$, we have Jacobian matrix $J(\mathbf{x}_i) = \begin{bmatrix} x_i^1 & 0 & 0 & 1 & 0 & 0 \\ 0 & x_i^2 & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i^3 & 0 & 0 & 1 \end{bmatrix}$. It can be verified that the rows of $B_2 = B([1, 4, 8, 11, 15, 18, 22], :)$ constitute the nonzero unique rows of B.

Analogous to Section 6.2 in the main paper, we use 2 categories of tests to evaluate the performances of different methods against outliers: 1) **Outlier test** and 2) **Occlusion + Outlier test**, as illustrated in Fig. S-1. Two shapes[1], a horse and a dinosaur, as shown in the left column of Fig. S-1, are used as the prototype shapes, respectively.



(a) (b) (c) (d) (e)

Figure S-1: (a) The prototype shapes. Examples of model and data point sets in the (b)∼(c) outlier and (d)∼(e) occlusion+outlier tests.

The average matching errors over 100 random trials by all the competing methods in the 2 categories of tests are shown in Fig. S-2, where error is defined as mean of the Euclidean distances between the warped ground truth model inliers and their corresponding data inliers. It can be seen that our method performs significantly better than others, and its matching error keeps almost unchanged with the increase of severity of disturbances. This demonstrates our method's strong robustness to outliers. Examples of matching results by different methods are shown in Fig. S-3.

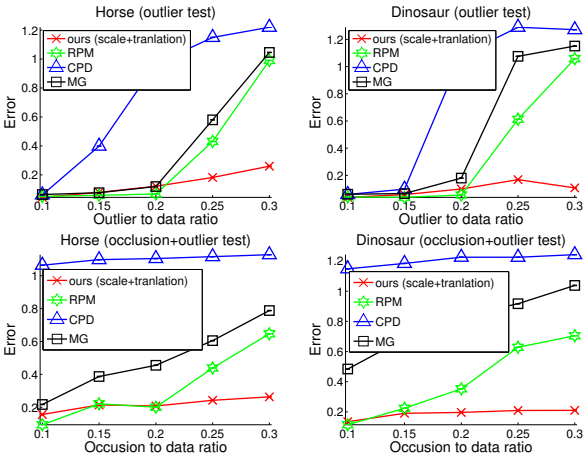


Figure S-2: Average matching errors over 100 random trails by different methods for the 2 categories of tests.

The average running time (in seconds) of different methods is listed in Table S-1.

Table S-1: Average running time of different methods (in seconds)

	ours (scale+translation)	RPM	CPD	MG
Outliers	57.8093	6.2224	0.1089	0.2196
Occlusion+Outliers	40.4617	4.6072	0.1111	0.2143

[1]These shapes can be downloaded at the AIM@SHAPE Shape Repository: *http://shapes.aimatshape.net/*.

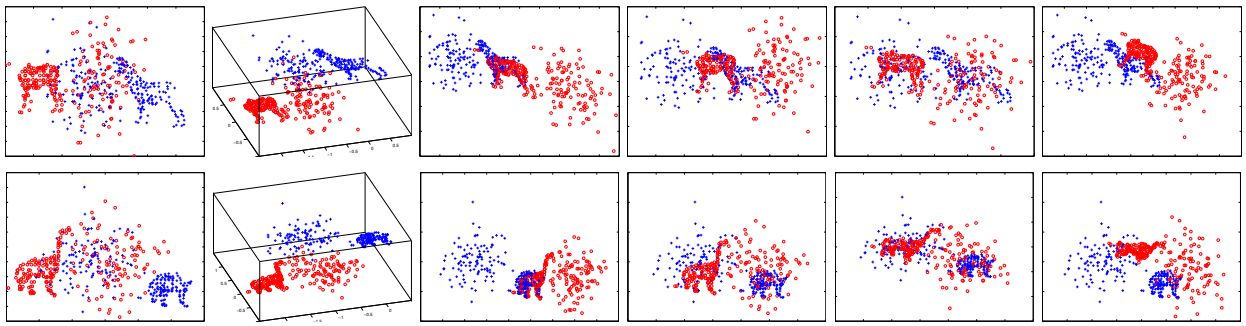


Figure S-3: Examples of matching results by different methods for the occlusion+outlier test. Top row: horse. Bottom row: dinosaur. (a) Before matching (front view); (b) before matching (side view); (c) ours; (d) RPM; (e) CPD; and (f) MG.