Coverage Analysis for Wireless Sensor Networks

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Abstract. The coverage problem in wireless sensor networks (WSNs) is to determine the number of active sensor nodes needed to cover the sensing area. The purpose is to extend the lifetime of the WSN by turning off redundant nodes. In this paper, we propose a mathematical model for coverage analysis of WSNs. Based on the model, given the ratio of the sensing range of a sensor node to the range of the entire deployment area, the number of the active nodes needed to reach the expected coverage can be derived. Different from most existing works, our approach does not require the knowledge about the locations of sensor nodes, thus can save considerably the cost of hardware and the energy consumption on sensor nodes needed for deriving and maintaining location information. We have also carried out an experimental study by simulations. The analytical results are very close to the simulations results. The proposed method can be widely applied to designing protocols for handling sensor deployment, topology control and other issues in WSNs.

1 Introduction

Technology advances in sensors, embedded systems, and low power-consumption wireless communications have made it possible to manufacture tiny wireless sensors nodes with sensing, processing, and wireless communication capabilities. The low-cost and low power-consumption sensor nodes can be deployed to work together to form a wireless sensor network. The sensor nodes in a sensor network are able to sense surrounding environment and carry out simple processing tasks, and communicate with the neighboring nodes within its transmission range. By means of the collaboration among sensor nodes, the sensed and monitored environment information (e.g. temperature, humidity) is transmitted to the base station for processing.

A large-scale wireless sensor network can consist of tens of thousands of tiny sensor nodes, with high density of sensors up to 20 nodes/m3. The high density of sensors may result in comparatively large energy consumption due to conflict in accessing the communication channels, maintaining information about neighboring nodes, and other factors. A widely-used strategy for reducing energy consumption while at the same time meeting the coverage requirement is to turn off redundant sensors by scheduling sensor nodes to work alternatively [1,2]. The coverage problem in wireless sensor networks (WSNs) is to determine the number of active sensor nodes needed to cover the sensing area.

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A broadly-used strategy [3][4][5][6][7][8] is to determine the active nodes by using the location information about the sensor nodes and their neighborhood. However, relying on complicated hardware equipment such as GPS (the Global Positioning System) or the directional antenna will greatly increase the hardware cost of sensor nodes and energy consumption; at the same time, the message transmission for and calculation of the locations and directions will also consume the energy of a node. Therefore, it is desirable for a solution to the coverage problem not to depend on any location information.

In this paper, we propose a mathematical model for coverage analysis of WSNs without requiring the use of location information. Based on the model, given the ratio of the sensing range of a sensor node to the range of the entire deployment area, the number of the active nodes needed to reach the expected coverage can be derived. The proposed analytical method is based on the random deployment strategy, which is the easiest and cheapest way for sensor deployment [10]. Comparing with similar work [13] using theoretical methods for coverage analysis without the use of location information, which is a special case of our work. It means our work is more general than [13].

Most applications may not require the maximal area coverage, and a small quantity of blind points generated at certain intervals can be accepted. If the working nodes in a sensor network can maintain a reasonable area coverage, most applications can realize. Coverage can be regarded as one quality of service of a sensor network to evaluate its monitoring capability [9]. If the coverage fraction is below certain threshold, the sensor network will be thought unable to work normally. So, it will be very significant to propose a simple method that can, in a statistical sense, evaluate the coverage fraction which meets the coverage requirement in application without depending on location information. This paper provides such a solution.

The rest of the paper is organized as follows. We introduce related work in section 2. In section 3, we present sensor network models and preliminary definitions. In section 4, we analyse the relationship between the ratio of the sensing range of a sensor node to the range of the entire deployment area and the coverage fraction. Numeric results and simulation results are provided in section 5.

2 Related Work

Coverage is one of the important issues in sensor networks. Because of different applications of sensor networks, maybe there are different definitions of coverage. We argue, in the case of K-cover, coverage in sensor networks can be simply described as: any point in the coverage area lies within the sensing range of at least K sensor nodes. Obviously, K is bigger than or equal to one. Wireless sensor networks are usually characterized by high density of sensor nodes and limited node energy. With the desired coverage fraction being guaranteed, working nodes density control algorithm and node scheduling mechanism are utilized to reduce energy cost and thus extend sensor network lifetime.

In [3] and [4], an approach is proposed to compute the maximal cover set: all the sensor nodes are divided into n cover sets which do not intersect one another, and the sensor nodes in each cover set can perform independently the task of monitoring the desired area; sensor nodes in all the cover sets take turns at performing the monitoring task. In [3], Slijepcevic et al. have proved that the calculation of the maximal cover set is as NP-complete problem. The two algorithms proposed in [3] and [4] are both centralized ones, so they are not suitable for the case in which there is a large quantity of sensor nodes. In addition, both the two algorithms have to rely on the location information of sensor nodes in reckoning cover sets.

In [1], Tian et al. propose a distributed coverage algorithm based on a node scheduling scheme. The off-duty eligibility rule proposed in this algorithm, relying on the geographical information of sensor nodes and AOA (Arrival of Angle) obtained through the directional antenna, can reckon the coverage relation between one node and its neighbors and then select working nodes. Obviously, sensor networks relying on GPS or AOA information are characterized by high cost and high consumption of energy. In addition, the off-duty eligibility rule fails to consider the problem that excessive overlap may be formed so that the number of working nodes selected becomes very large to cause extra energy consumption. In [11], it has been proved that this algorithm based on a node scheduling scheme is low-effective.

In [13], Gao et al. propose a mathematical method, which does not rely on location information, to describe the redundancy. According to this method, one sensor node can utilize the number of neighbors within its sensing range to calculate its own probability of becoming a redundant node. Since there is no need to be equipped with GPS or directional antenna, it is possible to get the cost of sensor nodes under control. In addition, it becomes unnecessary to derive location information through the exchange of message, and thus the energy consumption for communication in sensor networks is reduced. However, for most sensor nodes, the sensing hardware and the communicating hardware are two fully independent parts, and the communicating range is always not equal to the sensing range. Therefore, some specialized parts are needed to judge the number of neighbors within the sensing range.

As the above analysis suggests, most previously proposed coverage algorithms rely on outside equipment like GPS, directional antenna or positioning algorithm, etc. In this case, both the cost and the energy consumption are increased; in the mean time, some problems remain unsolved, e.g. GPS-based protocols have to correct some mistakes made in calculating location information; the work of GPS-based systems is not reliable in indoor environment, and thus some other positioning systems need to be deployed. For some positioning algorithms, each node needs to exchange a large quantity of information with the beacon node to calculate its location, and this will also result in high consumption of power. In [14], Stojemenvic makes a comprehensive analysis on location-based algorithms, and locations out that obtaining and maintaining location information will cause great consumption of energy.

In this paper, we provide an effective mathematical method to evaluate the number of nodes needed to reach the expected coverage fraction. In this method, only if the proportion of the node's sensing range to the range of the deployment area C is known, the relation between the number of sensor nodes in C and the expected coverage fraction can be derived by simple calculation. Therefore, our approach is applicable to many cases. It can be easily adopted in handling the problems of sensor deployment, topology control, etc.

3 Models and Assumptions

In this section, we first introduce two methods used in our research: the deployment method and the sensing method. Then we will give a few definitions to simplify the analytical process in Part 4.

3.1 Deployment Model

In [10], the commonly used deployment strategies are studied: random deployment, regular deployment, and planned deployment. In the random deployment strategy, sensor nodes are distributed with a uniformly distribution within the field. In the regular deployment strategy, sensors are placed in regular geometric topology such as a grid. In the planned deployment strategy, the sensors are placed with higher density in areas where the phenomenon is concentrated. In the planned deployment method, although sensors are deployed with a non-uniform density in the whole deployment area, however in a small range, sensors are approximately deployed randomly. In this sense, our analytical results of random deployment are also applicable to planned deployment.

The analysis in this paper is based on the random deployment strategy, which is reasonable in dealing with the application scenario in which priori knowledge of the field is not available. For convenience, we assume that sensor nodes are placed in a two-dimensional circular area C with a radius of R. Actually, we are not concerned about the shape of the deployment area, which can be circular or square, and the area C can represent a subset of the whole deployment area or represent the whole deployment area. Our research focuses on how to obtain the number of nodes required by C with the coverage of sensor network being guaranteed. We assume that sensor nodes are uniformly and independently distributed in the area C, and no two sensors can be deployed exactly at the same location.

3.2 Sensing Model

The analysis in this paper is based on Boolean sensing model, which is broadly adopted in the study of sensor networks [1][2][12]. In the Boolean sensing model, each sensor has a fixed sensing range. A sensor can only sense the environment and detect events within its sensing range. And in this paper all sensors are supposed to have the same sensing rang and the sensor's sensing range $r \le R$. A point is covered if and only if it lies within at least one sensor's sensing range. So, the deployment area is partitioned into two regions, the covered region and the vacant region. An arbitrary point in the covered region is covered by at least one sensor node, while the vacant region is the complement of the covered region. Actually, some applications require a higher degree of accuracy in detecting objects, so an arbitrary point in the covered region has to lie within the sensing ranges of k nodes at the same time. The analytical results in this paper, however, can be easily extended into K-coverage.

3.3 Related Definitions

To facilitate later discussion, we introduce the following definitions:

Definition 1: Neighboring area. For an arbitrary point $(x, y) \in C$, its neighboring area is defined as

$$\aleph(x, y) = \left\{(x', y') \in C \mid \forall\left((x' - x)^2 + (y' - y)^2 \le r^2\right)\right\}$$

Definition 2: The central area $C^{'}$**.** For $C^{'}$, we have $C^{'} \subset C$. And for an arbitrary point $(x, y) \in C^{'}$, there is

$$x^2 + y^2 < (R - r)^2$$

Definition 3: Expected coverage fraction, denoted as q. Expected coverage fraction of a sensor network is defined as the expected proportion of the covered region to the whole deployment region. For example, an application requires the coverage fraction reach 85 percent of the whole region, and the expected coverage fraction equals 0.85. If the expected coverage fraction is known, it can be used to calculate the number of nodes needed to cover the deployment area.

As shown in Figure 1, for an arbitrary point, its neighboring area is actually the overlapped area of the circle centered at the point with a radius of r and the area *C***.** The central area C', which and *C* are circles centered at the same point, has a radius of $R - r$. Obviously, the neighboring area of every point in $C^{'}$ is the same**,** and its value is πr^2 . For any point in $C - C^{'}$, its neighboring area is in inverse proportion to its distance away from the centered point of C, and is less than πr^2 .

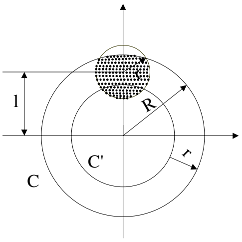

Fig. 1. Illustration of analysis

4 Analysis for Coverage

For an arbitrary point $(x, y) \in C$, if there exists at least one sensor node in its neighboring area, the point is covered. Since the sensors in *C* are distributed randomly and uniformly, the probability that an arbitrary node falls on the point's (x, y) neighboring area is $p = \aleph(x, y)_{area} / C_{area}$.

Assume that m sensor nodes are deployed randomly in *C*. In the case of single-cover, the probability that an arbitrary point is covered is equal to the one that at least one sensor node falls on its neighboring area, namely,

$$\begin{aligned} p_{\{(x,y)\in C\}} &= c_m^1 p(1-p)^{m-1} + C_m^2 p^2(1-p)^{m-2} + \cdots\cdots + C_m^m p^m \\ &= \sum_{n=1}^{m} C_m^n p^n (1-p)^{m-n} \\ &= 1 - (1-p)^m \end{aligned} \tag{1}$$

Hence, for any two points in C, if the size of their neighboring area is the same, the probability of being covered is equal to each other. For $\forall(x,y)\in C'$, it's neighboring area $\aleph(x,y)_{area}=\pi r^2$, so the probability that an arbitrary node falls on certain point's neighboring area in *C'* is $p=\aleph(x,y)_{area}/C_{area}=\pi r^2/\pi R^2$. According to Formula 1, if there are m sensor nodes randomly deployed in *C*, for $\forall(x,y)\in C'$, the probability of being covered is

$$p_{\{(x,y)\in C'\}}=\sum_{n=1}^{m}C_m^n\left(\frac{r}{R}\right)^{2n}\left(1-\frac{r^2}{R^2}\right)^{m-n} \tag{2}$$

For each point that lies within the marginal region $C-C'$ of *C*, its neighboring area is less than πr^2; and especially for each point on the edge of *C*, its neighboring area is the smallest, and thus its probability of being covered is also the smallest. Assume the probability that each point on the edge of C is covered is $p_{\min}$, and then it is obvious that $p_{\min}\le p_{\{(x,y)\in C\}}\le p_{\{(x,y)\in C'\}}$. When $R\gg r$, the area of $C-C'$ can be ignored in calculation. In this case, it can be approximately concluded that for an arbitrary point in *C*, the probability of being covered is the same: $p_{((x,y)\in C)}\approx 1-\left(1-\frac{r^2}{R^2}\right)^m$. Since the probability that each point in *C* is covered is $1-\left(1-\frac{r^2}{R^2}\right)^m$, the expected coverage fraction can be: $q=1-\left(1-\frac{r^2}{R^2}\right)^m$.

When the proportion of r to R is small enough that it cannot be ignored, to use Formula 2 to calculate the expected coverage fraction can lead to an error that is beyond tolerance. Therefore, in order to have an accurate evaluation of q, we have to compute the average probability of being covered for all the points in *C*. As shown in Figure ( ), there is one point (x_1,y_1), and l denotes the distance between this location and the center of Circle *C*, and $l>R-r$. Then the value of point's neighboring area $\aleph(x_1,y_1)_{area}$ is the area of the shadowed region:

$$\begin{aligned}\aleph(x_1,y_1)_{area}&=2\left(\int_{l-r}^{\frac{R^2-r^2+l^2}{2l}}\sqrt{r^2-(y_1-l)^2}\,dy_1+\int_{\frac{R^2-r^2+l^2}{2l}}^{R}\sqrt{R^2-y_1{}^2}\,dy_1\right)\\&=\frac{1}{2}\pi\left(r^2+R^2\right)+r^2\arcsin\frac{R^2-r^2-l^2}{2lr}+\frac{R^2-r^2-l^2}{2l}\sqrt{r^2-\frac{\left(R^2-r^2-l^2\right)^2}{4l^2}}\\&\quad-R^2\arcsin\frac{R^2-r^2+l^2}{2lR}-\frac{R^2-r^2+l^2}{2l}\sqrt{R^2-\frac{\left(R^2-r^2+l^2\right)^2}{4l^2}}\end{aligned} \tag{3}$$

From Formula 3, we can derive a common expression of the neighboring area of any point in $C-C'$. By the operation of integral calculus**,** we can calculate the average size of neighboring area of all the points in $C-C'$**,** denoted as $\overline{\aleph(x,y)}_{C-C'}$:

$$\overline{\aleph(x,y)}_{C-C'} = \iint\limits_{c-c'} \aleph(x_1,y_1)_{area} d_\sigma \Big/ \pi\left[R^2-(R-r)^2\right]$$
$$= 2\pi\int_{R-r}^{R} l\aleph(x_1,y_1)_{area} d_l \Big/ \pi\left[R^2-(R-r)^2\right] \tag{4}$$

And the average neighboring area of all the points in *C* is:

$$\overline{\aleph(x,y)}_{area} = \left[2\int_{R-r}^{R} \aleph(x_1,y_1)_{area} d_l + r^2\times\pi(R-r)^2\right] \Big/ R^2 \tag{5}$$

Hence, for all the points in *C*, the average probability of being covered, i.e. the expected coverage fraction in *C* is:

$$q = 1-(1-\frac{\overline{\aleph(x,y)}_{area}}{\pi R^2})^n \tag{6}$$

Our previous discussion only involves the single-cover. In the case of k-cover, there are at least k nodes in the neighboring area of an arbitrary point in the covered region. The probability of being covered is

$$p_{\{(x,y)\in C\}} = c_m^k p(1-p)^{m-k} + C_m^{k+1}p^2(1-p)^{m-k+1} + \cdots\cdots + C_m^m p^m$$
$$= \sum_{n=k}^{m} C_m^n p^n (1-p)^{m-n} \tag{7}$$

Once the radius of *C* (denoted as R) and the node's sensing range (denoted as r) are determined, the probability that an arbitrary point in *C* is covered relates only to its neighboring area. Therefore, the above discussion based on the 1-cover is still applicable in dealing with multi-cover.

5 Analysis and Evaluation

5.1 Numerical Results

As Table 1 shows**,** the larger the proportion of r to R is, the smaller the size of the average neighboring area of all the points in C will be. When R = r, the proportion of the average neighboring area to the maximal neighboring area is only 0.5781. But when r<<R, e.g. r/R = 0.01, the proportion, as shown in Table 1, is approximately equal to 1. In this case, we can use Formula 2 to approximately compute the expected coverage fraction in *C*.

Table 1.

r_s/R	1	9/10	8/10	7/10	6/10	5/10	4/10	3/10	2/10	1/10	1/100
$\overline{\aleph}/\aleph_{max}$	0 5865	0 6259	0 6660	0 7066	0 7477	0 7891	0 8309	0 8730	0 9152	0 9576	0 9958

5.2 Simulation Methodology

Our simulation, based on MATLAB, gets started with the production of the deployment region *C* consisting of pixels. In order to make sure that the experimental

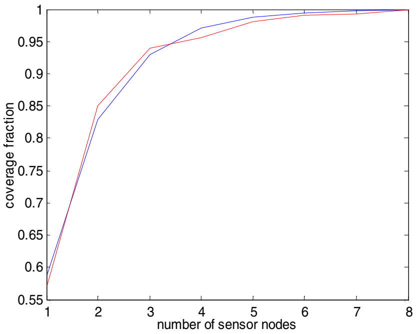

Fig. 2. 1-cover & r/R = 1

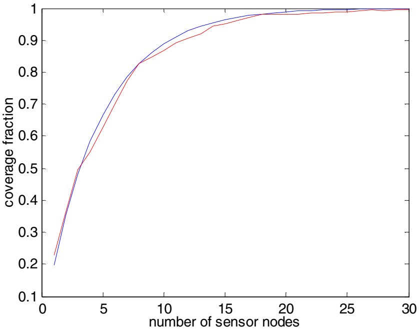

Fig. 3. 1-cover & r/R = 0.5

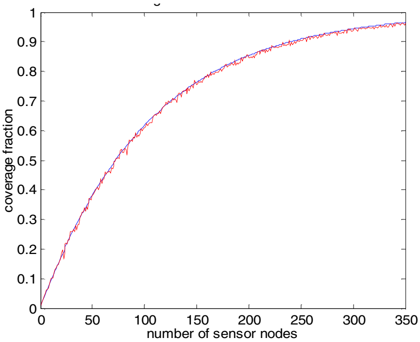

Fig. 4. 1-cover & r/R = 0.1

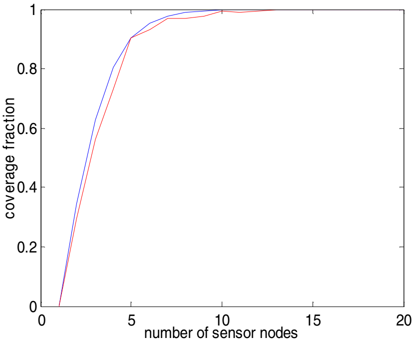

Fig. 5. 2 - cover & r/R = 1

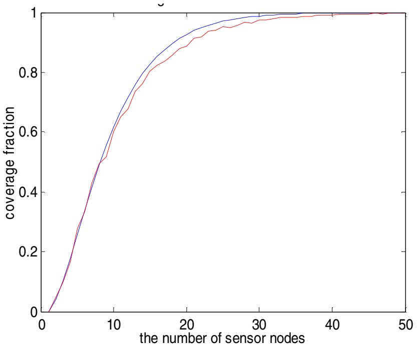

Fig. 6. 2-cover & r/R = 0.5

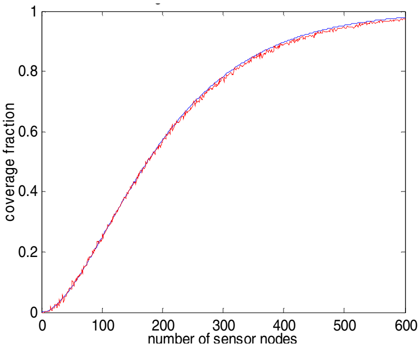

Fig. 7. 2-cover & r/R = 0.1

result is accurate, the region *C*, which is made up of 19634803 pixels, has a radius of 5000 pixels. According to N, the number of the deployed sensor nodes, we randomly select N locations and let them distributed uniformly and independently in *C*. Each pixel is defined as a structure, and for each pixel we count the number covered by sensor nodes. It is obvious that the percentage of the coverage fraction is equal to the proportion of the number of the covered pixels to the number of the whole pixels.

5.3 Simulation Results

To fully test the accuracy of our analytical results, the experiment simulates 1-cover and 2-cover in the three cases in which the proportion of the sensing range r to the deployment range R is assumed to be 1, 0.5 and 0.1 respectively. All the results reported are averages of 50 simulation runs.

From figure 2 to figure 7 shows the simulation results of coverage fraction as a function of nodes number. As figure 2 shown, we observe that if there exist more than four nodes in deployment area C, the coverage fraction derived from simulation equals 0.95611 and the expected coverage fraction equals 0.97077. And when 6 nodes deployed in C area, the coverage fraction derived from simulation equal 0.99145 and the expected coverage equal 0.995. So it suggests that our analytical results are very similar to the simulation results.

As figure 2 to figure 7 shown, Under the random deployment, the deviation between the coverage fraction derived from our theoretical analysis and that obtained from the simulations is no larger than 5% of the analytical value; Given a coverage fraction, the deviation between the number of working nodes derived from analysis and that obtained from the simulation is less than 10% of the analytical value**.** This suggests that our results are identical to the experimental results.

6 Conclusion and Future work

In this paper, we proposed a mathematical method for coverage analysis of WSNs. Using the method, given the ratio of the sensing range of a sensor node to the range of the entire deployment area, the number of the active nodes needed to reach the expected coverage can be derived. The main contribution of this paper lies in a simple and effective approach to solving the coverage problem without the need of sensor nodes' location information and its potential application in developing heuristic algorithms for node scheduling, clustering, and other functions.

Our future work includes applying the proposed method to design energy efficient protocols for various WSN functions and extending our model to cover different node distribution and sensor network models.

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