

# ORTEGA: An Efficient and Flexible Online Fault Tolerance Architecture for Real-Time Control Systems

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# Outline

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- Motivation and related work
- ORTEGA goals
- ORTEGA architecture
- Details of ORTEGA designs
- Implementation and evaluation
- Demo



# Motivations

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- Cyber-Physical Systems
  - Real-world systems involves not only computer science, but knowledge related to various disciplines.
  - Not only the computer system becomes more complex, the complexity of integrated system (i.e. the cyber-physical system) grows even faster.
  - Major challenge: how to let engineers of drastically different backgrounds collaborate with each other?



# Motivations

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- Control Systems

$$\dot{x} = Ax + Bu$$

- Conventional analog control systems  $u = -Kx$

- Digital control systems

$$x(kh + h) = e^{Ah}x(kh) + \left( \int_0^h e^{As} ds \right) Bu(kh)$$

- Computer Systems

$$u(kh) = -Kx(kh)$$

- Real-time scheduling
- Fault tolerance
- Reliable/online software upgrade

- We need to design a framework so that computer engineers and control engineers can easily collaborate and integrate their knowledge

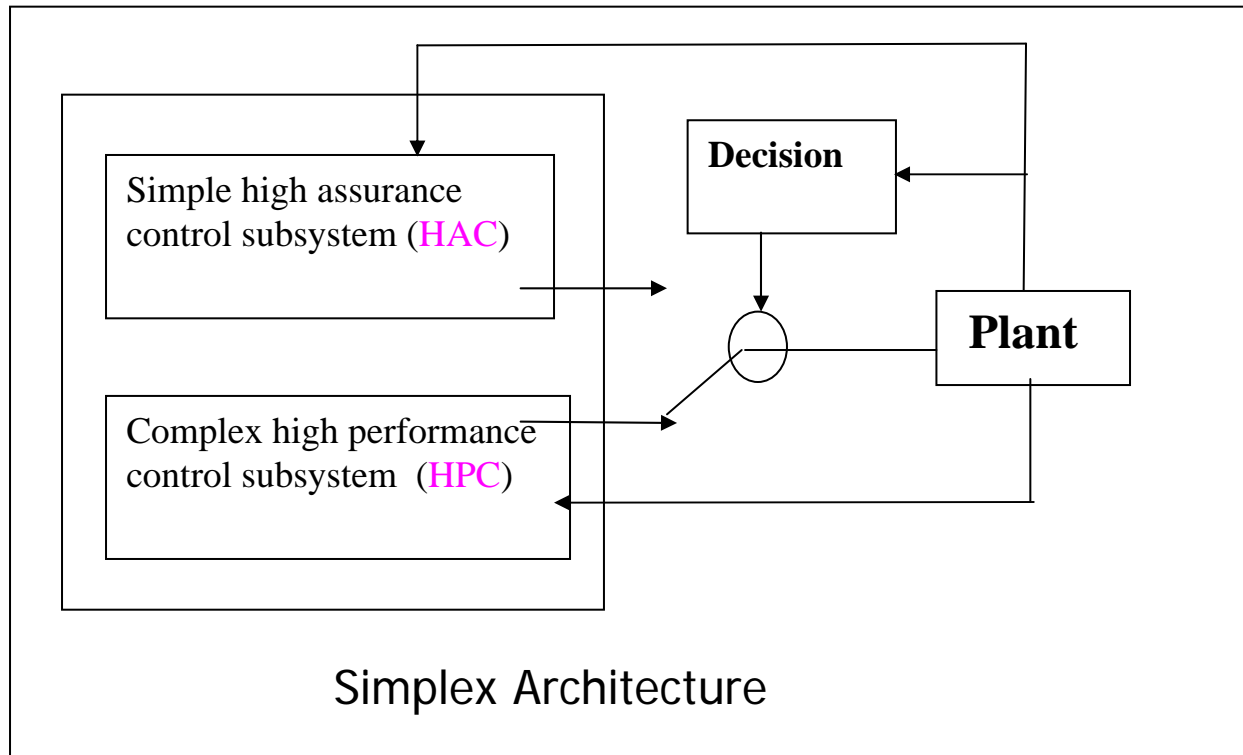


# Related work: Simplex architecture

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- Demand:
  - Low cost development of upgraded control systems for mission critical control applications
    - instead of multi-versioning, just develop one version
    - Focus on the control theories
  - Runtime upgrade/testing of the single version buggy new system.
- Applications:
  - Aircraft control (F-16, Seto et. al, 2000)
  - Submarine control (NSSN, new attack submarine program at US navy)

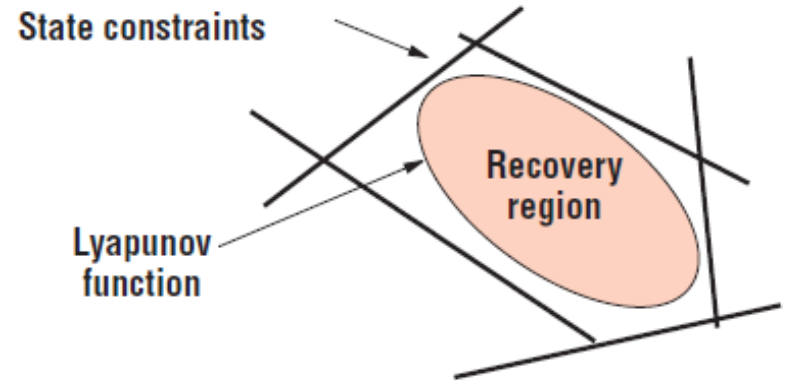
# Simplex for real-time control



# Simplex for real-time control

Given LTI control system:

$$\begin{aligned}\dot{x} &= \bar{A}x + Bu \\ &= \bar{A}x - BKx = Ax\end{aligned}$$

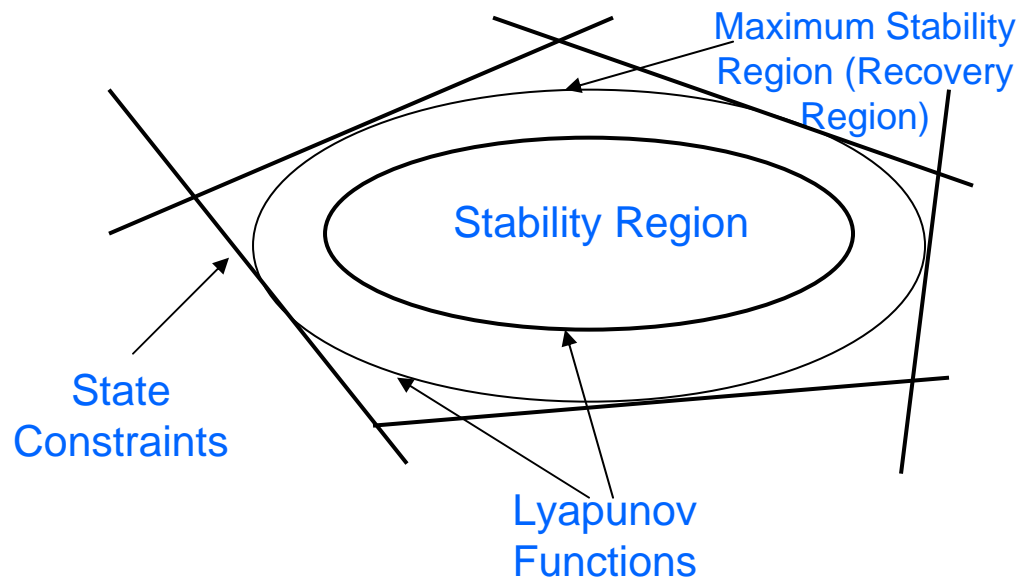


The above LTI control system is stable iff there exists a  $P > 0$ , such that the Lyapunov function

$$x^T (A^T P + PA)x < 0$$

The solution ellipsoid is maximized by minimizing  $\log \det P^{-1}$

# Simplex for real-time control



We can choose smaller solution ellipsoid (i.e.  $x^{TP}x < x^{TPmax}x$ ) to leave margins to guard against model/actuator/measurement errors.





# Drawbacks of Simplex

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- P1: Lack of Efficiency

- Analytically redundant high assurance controller (HAC) runs in parallel with complex controller (HPC)
  - Lowers system performance, increase operating costs
  - Limits the application of Simplex in only safety-critical domains

- P2: Lack of Flexibility

- Enforces the same execution period on HAC and HPC
  - In practice, different controllers may use different periods for different performance considerations
  - For example: fast HAC recovery

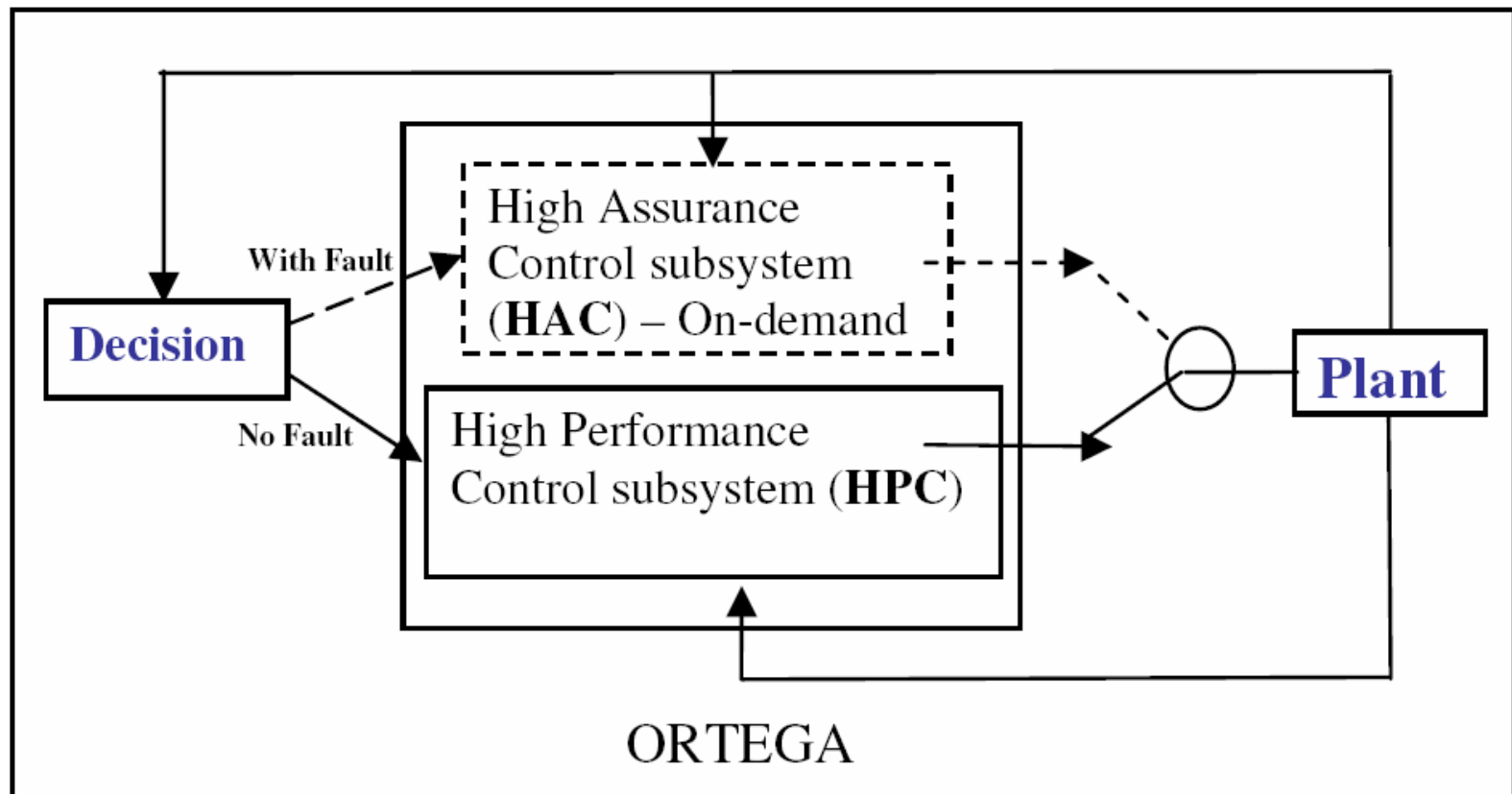


# Design goals of ORTEGA

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- On-demand Real-Time Guard (ORTEGA)
  - A new efficient fault tolerance software architecture designed for real-time control systems
- More efficient resource usage (P1)
  - Through on-demand real-time recovery
- Flexible design (P2)
  - Allows HAC and HPC to run at different rates
  - Through new design and schedulability analysis
- Applicable to a wider range of real-time control systems

# ORTEGA Architecture





# On-demand execution of HAC

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- At any time, only one of the HAC or HPC is running to control the plant
- Decision module (DM) uses a mutex semaphore to control which of the HAC and HPC is running
  - When the HPC is running well, the HAC blocks on the semaphore;
  - Only when a fault is detected in the HPC, the DM releases the semaphore to allow HAC to take over
- Decision logic is based on stability regions
  - Determined through Linear Matrix Inequality theory
  - Details later



# CPU savings of ORTEGA

HPC's timing parameters:  $\{C^p, T^p\}$ ; HAC's timing parameters:  $\{C^a, T^a\}$ ;

$P_r$ : the percentage of time for recovery (HAC) during a total time of  $T$

- Total CPU resource usage under Simplex

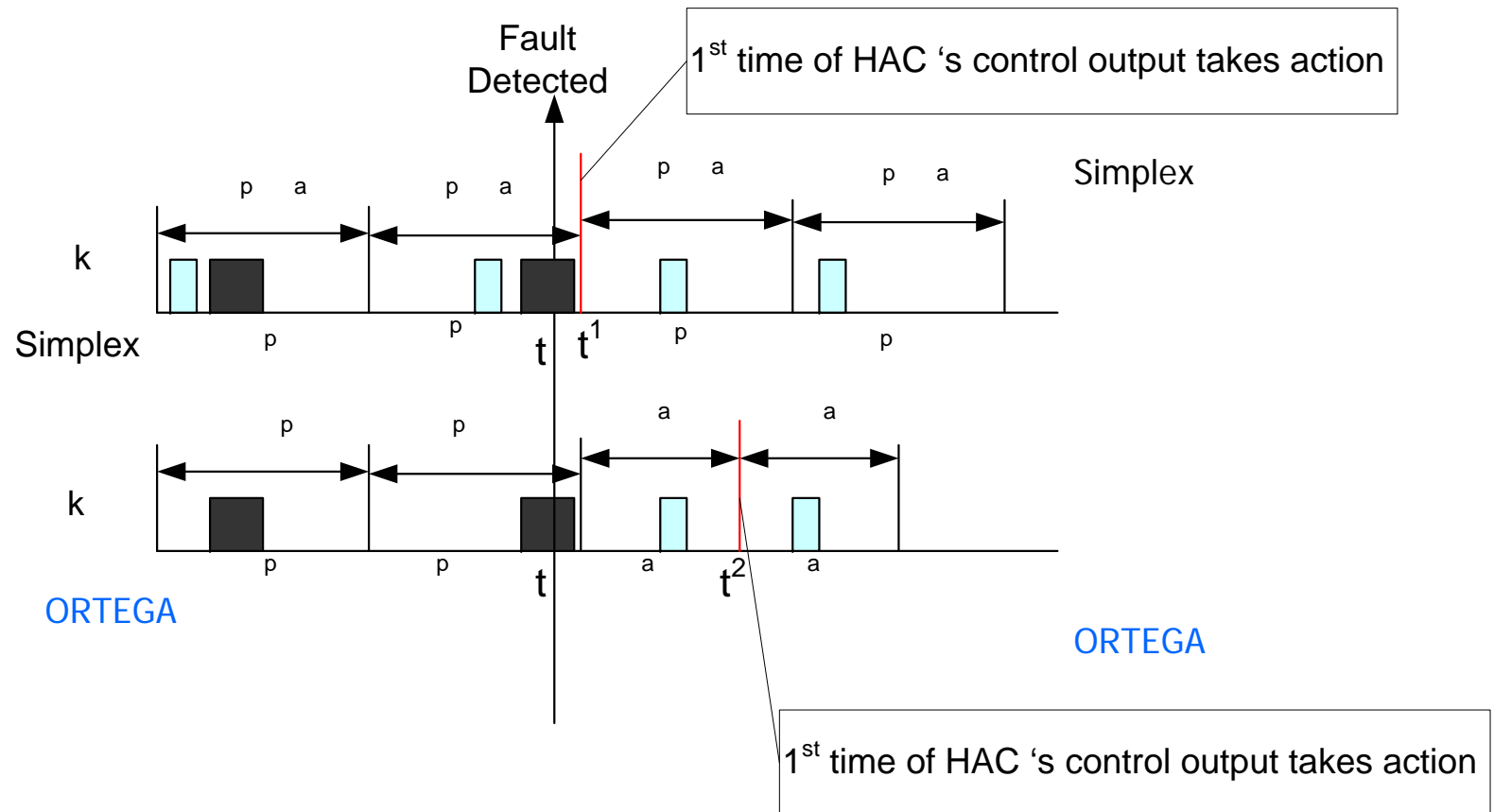
$$R_{Simplex} = (1 - P_r) \cdot \left( C^a \cdot \left\lceil \frac{T}{T^a} \right\rceil + C^p \cdot \left\lceil \frac{T}{T^p} \right\rceil \right) + P_r \cdot C^a \cdot \left\lceil \frac{T}{T^a} \right\rceil$$

- Total CPU resource usage under ORTEGA

$$R_{ORTEGA} = (1 - P_r) \cdot C^p \cdot \left\lceil \frac{T}{T^p} \right\rceil + P_r \cdot C^a \cdot \left\lceil \frac{T}{T^a} \right\rceil$$

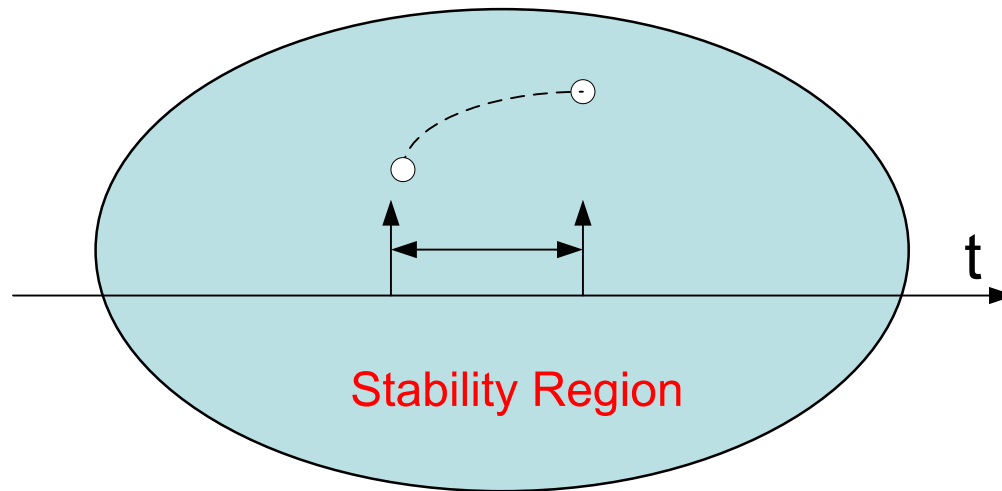
- CPU resource usage savings:  $(1 - P_r) \cdot C^a \cdot \left\lceil \frac{T}{T^a} \right\rceil$

# No Free Lunch: An extra period of delay



up to  $T^a$  incurred due to the on-demand execution of HAC

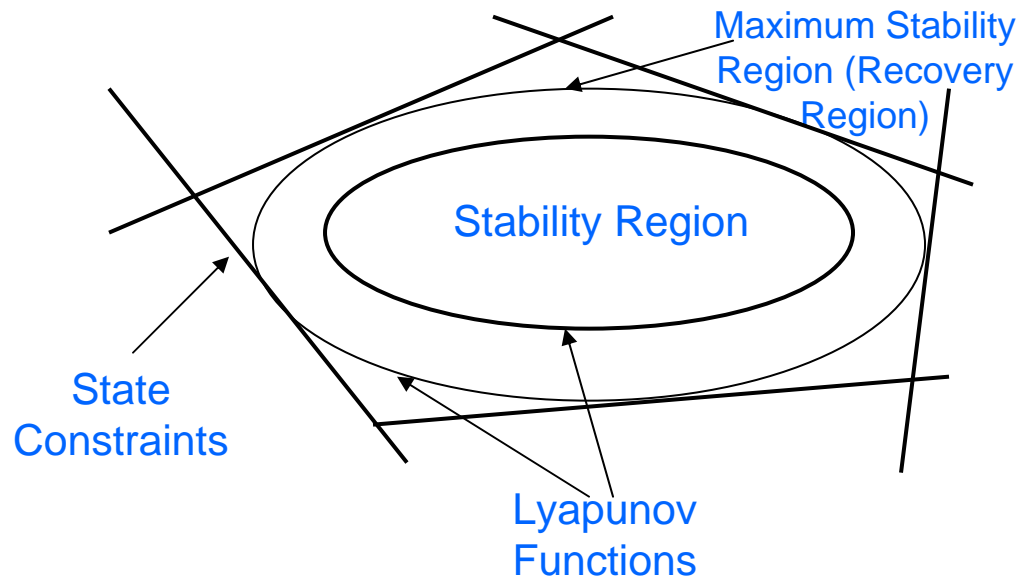
# Handle the extra delay by state projections



Resource usage reduction v.s. extra delay :

- (1) Extra delay causes disturbances when fault occurs (infrequent)
- (2) But the gain in resource usage is large.

# Recovery region design



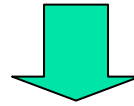
- The decision module uses recovery region to determine when to switch to HAC
- Recovery region is defined as the maximum region in which the HAC can make the plant stable



# Determine recovery region (1)

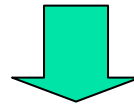
Digital controllers:

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t),$$



$$x(k+1) = F(h)x(k) + G(h)u(k),$$

$$u(k) = -Kx(k)$$



$$x(k+1) = \bar{F}x(k) \quad (*) \quad (\bar{F} = F - GK)$$

State constraints:

$$\alpha_m^T x \leq 1, \quad m = 1, \dots, q. \quad (1)$$

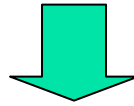
Stability region:

The discrete LTI control system is stable iff there exists a  $P > 0$ , such that  $\bar{F}^T P \bar{F} - P < 0$

# Determine recovery region (1)

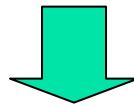
Digital controllers:

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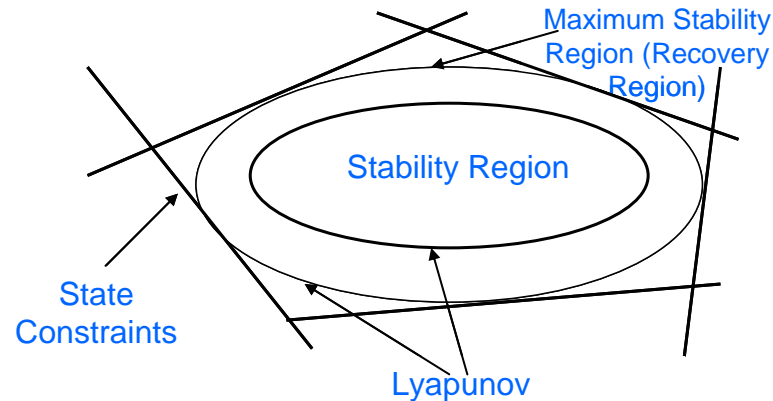
State constraints:

$$\alpha_m^T x \leq 1, \quad m = 1, \dots, q. \quad (1)$$

Stability region:

Stability region of the system with respect to  $P$  is defined as  $\{x \mid x^T P x < 1\}$ .

# Determine recovery region (2)



**Theorem:** Determine the maximum stability region of digital implemented closed loop system with constraints (1) can be transformed to the following MAXDET (LMI) problem.

$$\begin{array}{ll}
 \text{Maximize} & \log \det P^{-1} \\
 \text{s.t.} & P > 0, \\
 & \bar{F}^T P \bar{F} - P < 0, \\
 & \alpha_m^T P^{-1} \alpha_m \leq 1, \quad m = 1, \dots, q.
 \end{array}$$

Area of recovery region

Stability

State constraints

# Recovery region v.s. control loop period

**Stability Index  $A(T)$ :** Area of the maximum stability region

- It is a function of the control loop period  $T$ . The smaller the controller loop period, the larger the maximum stability region.

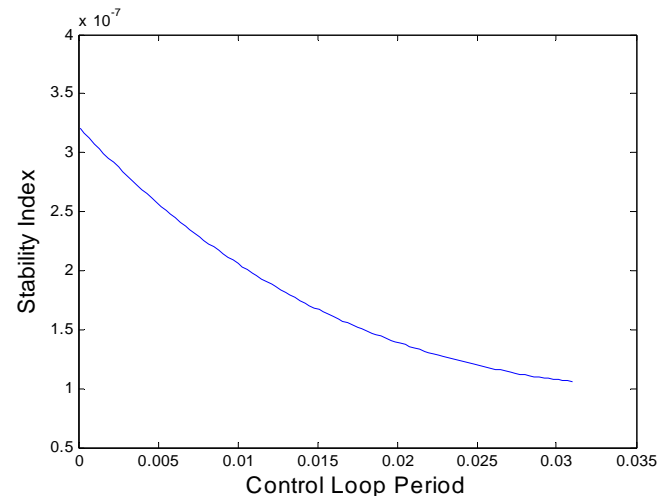
Example: an inverted pendulum

## System model

$$\dot{x} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -2.7528 & -10.9526 & 0.0043 \\ 0 & 28.5812 & 24.9179 & -0.0441 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 1.9432 \\ -4.4385 \end{pmatrix} u$$

## Controller

$$u(k) = -[5.7807, 42.2087, 14.0953, 8.6016]x(k)$$

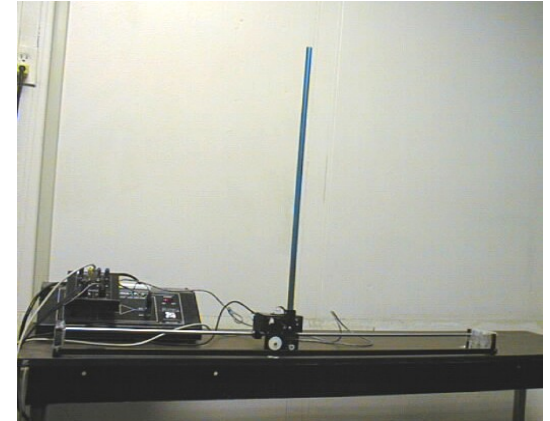


The smaller the period, the larger the recovery region.

ORTEGA allows larger recovery region (more flexible)

# Implementation and evaluation

- Inverted pendulum from Quanser
- CPU: Pentium II 350MHz
- OS: Linux kernel 2.4.18-3 with RMS
- HAC: field tested state feedback controller



## Evaluation of CPU savings

Table 1. Execution statistics for the non-faulty HPC and the HAC

Controller	Average Execution Time ( $\mu s$ )	Variance of Execution Time	Minimum Execution Time ( $\mu s$ )	Maximum Execution Time ( $\mu s$ )
HPC	2.6705	0.02181	2.3571	3.2857
HAC	1.1060	0.005812	0.9429	1.6371

- If HAC and HPC both run at 50Hz, ORTEGA's CPU saving is 29.29%
- If HAC runs at 50Hz, HPC runs at 20Hz, ORTEGA's CPU saving is 50.87%

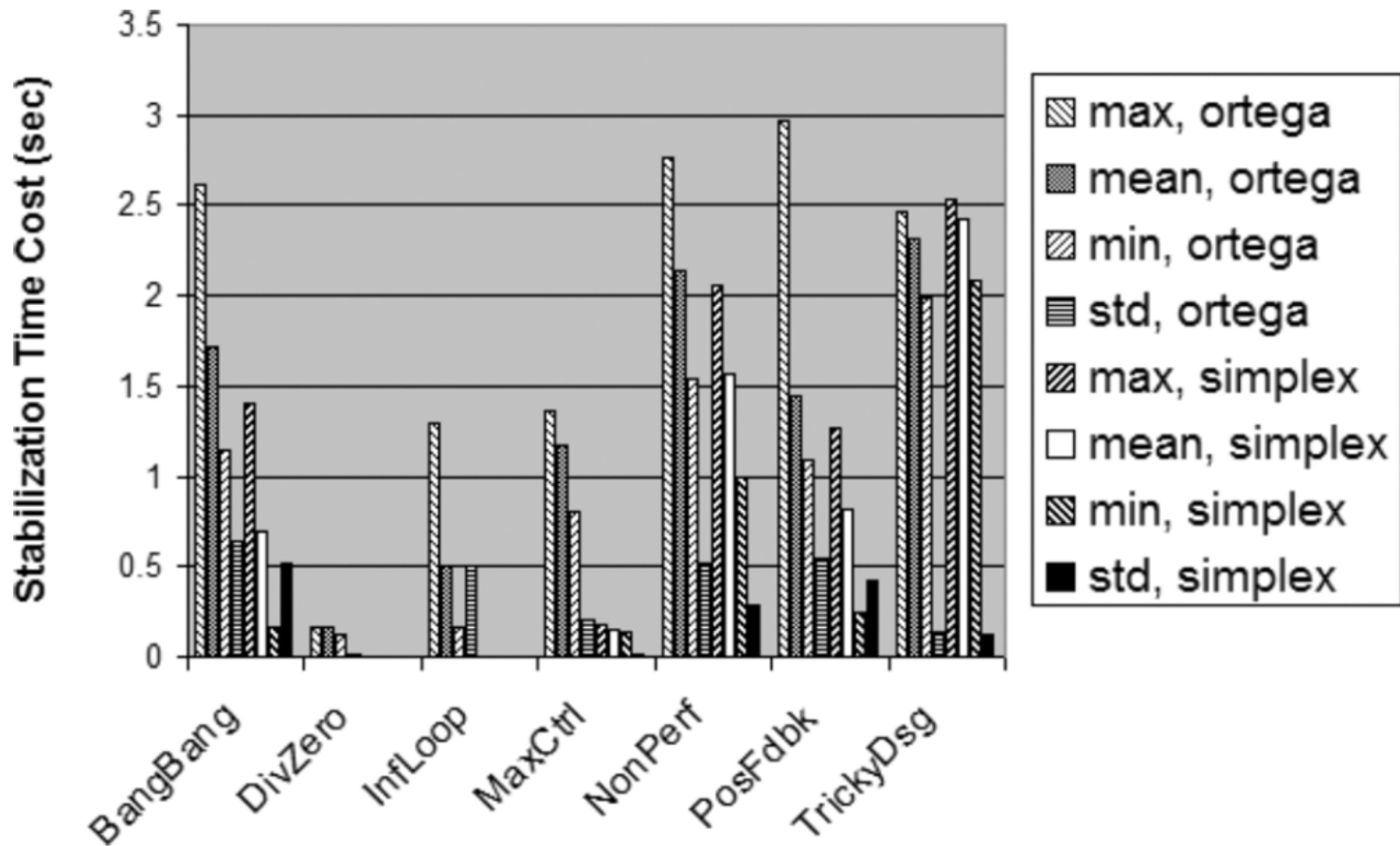


# Evaluation of fault tolerance

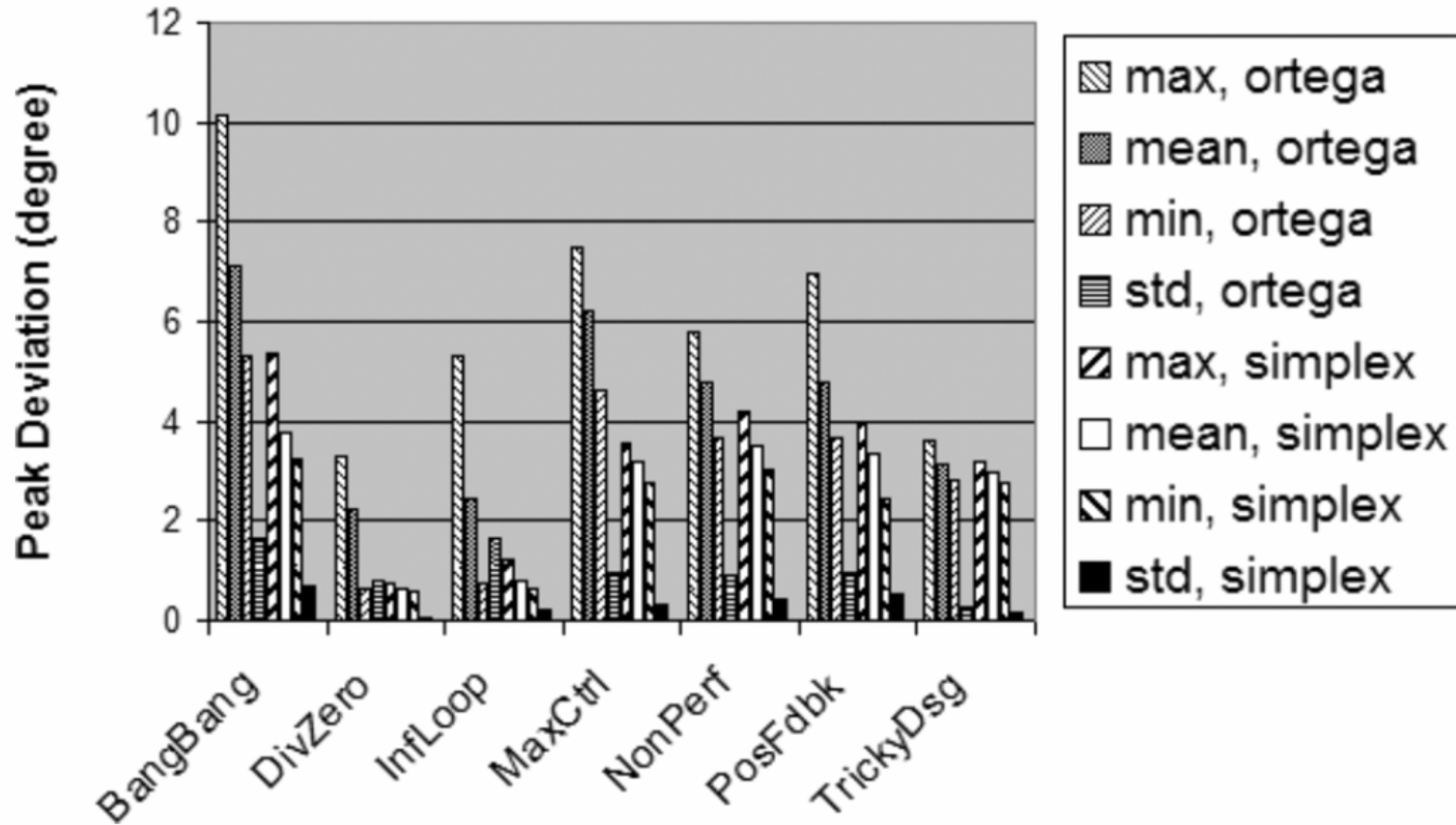
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- Infinite loop bug
- Non-performing bug
- Maximum control output bug
- Divided by zero bug
- Bang-Bang type bug
- Positive feedback bug
- Tricky design bug
- ...

# Evaluation of fault tolerance



# Evaluation of fault tolerance



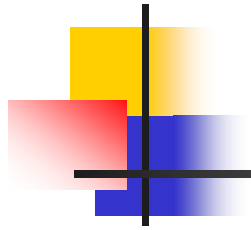




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*Thank You*

*Q&A*



# Backup Slides



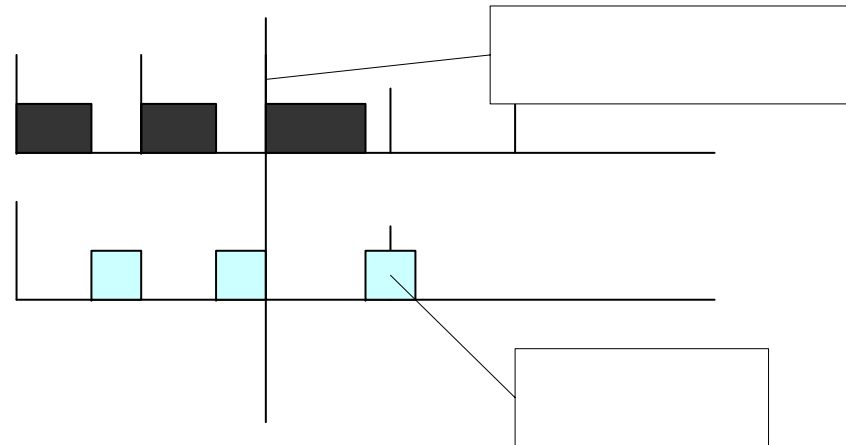
# Schedulability analysis of ORTEGA

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# Mode-Change Problem Incurred by Recovery

**Example:** Suppose one plant  $\tau_1^p : (C_1^p, T_1^p) = (3, 5)$ ;  $\tau_1^a : (C_1^a, T_1^a) = (4, 10)$  ;  
with another real time task  $\tau_2 : (C_2, T_2) = (6, 15)$ .

- Before the recovery at  $t=10$ ,  $\{\tau_1^p, \tau_2\} = \{(3, 5), \{6, 15\}\}$  is schedulable;
- After the recovery transition,  $\{\tau_1^a, \tau_2\} = \{(4, 10), \{6, 15\}\}$  is also schedulable;
- However, during the transition of recovery,  $\tau_2$  misses its deadline at  $t=15$ !



Unschedulable  
of tasks due to  
the recovery

Mode-change in fixed priority scheduling is a well-recognized difficult problem by the real-time community

# Schedulability Analysis

**Schedulability Analysis:** We adopt the work by Real and Crespo (2004)

**Idea:** Analyze the transitional scheduling overhead incurred by the recovery.

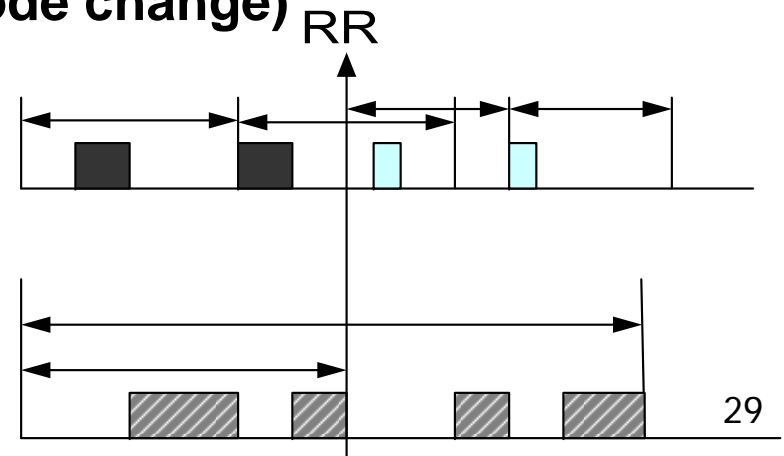
**(I) Schedulability analysis of steady state task set**

**(II) Schedulability analysis of old-mode tasks with transitional scheduling overhead (due to the mode change)**

$$w_i(x) = C_i + \left\lfloor \frac{x}{T_k^p} \right\rfloor C_k^p + \min \left( x - \left\lfloor \frac{x}{T_k^p} \right\rfloor T_k^p, C_k^p \right) + \left\lfloor \frac{w_i(x) - x}{T_k^a} \right\rfloor_0 C_k^a + \sum_{j < i, j \neq k} \left\lfloor \frac{w_i(x)}{T_j} \right\rfloor C_j .$$

**(III) Schedulability analysis of new-mode tasks with transitional scheduling overhead (due to the mode change)**

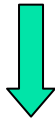
$$w_i = C_i + \left\lfloor \frac{w_i}{T_k^a} \right\rfloor C_k^a + \sum_{j < i, j \neq k} \left( \left\lfloor \frac{w_i}{T_j} \right\rfloor C_j \right) .$$



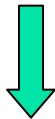
# Fault Tolerance and Scheduling Co-design

## -- one FT-enabled task case

Maximize the recovery region subject to schedulability constraint



Find the smallest (optimal) control loop period  $T_k^{*a}$ , s.t. the task set is schedulable under random recoveries



Given the schedulability test, we can use binary search algorithm to find  $T_k^{*a}$

**Example:** 3 tasks.  $\tau_1 = (2, 4)$  and  $\tau_3 = (3, 30)$  are ordinary real-time tasks.  $\tau_2$  is a FT-enabled task, with  $\tau_2^p = (2, 8)$ .

### Numerical Solution:

- (1) If  $C_2^a = 2.0$ , we have  $T_2^{*a} = 6.5 < T_2^p$ ;
- (2) If  $C_2^a = 1.5$ , we have  $T_2^{*a} = 4.5 < T_2^p$ ;
- (3) If  $C_2^a = 1.0$ , we have  $T_2^{*a} = 3.0 < T_2^p$ ;
- (4) If  $C_2^a = 0.5$ , we have  $T_2^{*a} = 2.5 < T_2^p$ .

## P2: Recovery Region for Digital Controllers

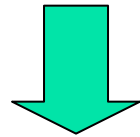
$$\frac{dx(t)}{dt} = Ax(t) + Bu(t), \quad \longrightarrow \quad x(k+1) = F(h)x(k) + G(h)u(k),$$

Sampling time  $h$ ,  
Zero-order hold

$$F(h) = e^{Ah}, \quad G(h) = \int_0^h e^{As} ds B.$$

**Controller**

$$u(k) = -Kx(k)$$



$$x(k+1) = \bar{F}x(k) \quad (\bar{F} = F - GK)$$

**Theorem (Lyapunov):** A discrete time LTI system shown above is stable iff there exists a matrix  $P > 0$ , such that

$$\bar{F}^T P \bar{F} - P < 0.$$

## Stability Region (Continued)

Stability region of the system  $x(k+1) = \bar{F}x(k)$ ,  
with respect to  $P$  is defined as:  $\{x \mid x^T P x < 1\}$ .

## Stability Region with Constraints

State constraints

$$a_i^T x \leq 1, \quad i = 1 \cdots l,$$

Control input constraints

$$b_j^T u \leq 1, \quad j = 1 \cdots r.$$

Can be combined in the  
closed loop system as

$$\alpha_m^T x \leq 1, \quad m = 1, \dots, \bar{m}. \quad (1)$$

**Lemma:** The stability region defined above satisfy constraints (1) iff  $\alpha_m^T P^{-1} \alpha_m \leq 1, \quad m = 1, \dots, \bar{m}.$