A Wavelet-based Edge Detection Method by Scale Multiplication

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Abstract

A wavelet-based multiscale edge detection scheme is presented in this paper. By multiplying the wavelet coefficients at two adjacent scales to magnify significant structures and suppress noise, we determined edges at the local maxima directly in the scale product after an efficient thresholding instead of first forming the edge maps at several scales and then synthesizing them together, which was employed in many multiscale techniques. It is shown that the scale multiplication achieves better results than either of the two scales, especially on the localization performance. Experiments on natural images are compared with the Laplacian of Gaussian (LOG) and Canny edge detection algorithms.

1. Introduction

Edge detection is an essential process in image analysis and many techniques have been proposed. Canny [3] evaluated the detectors by three criteria: good detection, good localization and low spurious response, and he showed that the optimal detector for an isolated step edge should be the first derivative of Gaussian. Besides the shape of the detector, another important problem is to set a proper detection scale. As suggested by Marr and Hildreth [2], multiple scales should be employed to describe the variety of the edge structures.

Canny [3] used a fine-to-coarse feature synthesis strategy to merge the multiscale edge information based on a set of predefined rules. Bergholm [4] combined the multiscale edges in a coarse-to-fine tracking manner. The RRES scheme of Lu and Jain [5] tends to be more complex with so many knowledge rules and continuous scale space. Considering that the synthesis of the multiscale edges is intricate and itself an ill-posed problem, Jeong and Kim [9] selected an optimal scale adaptively for each of the pixels by minimizing an objective function, but the results suffered from the complicated shape of the function and the sensitivity to the initial scale. Ziou and Tabbone [6] ran a subpixel Laplacian operator at two scales and then recovered the edges with four-step edge models. Park et al. [7] divided an image into several regions based on a discontinuity measure calculated over a window and then selected different resolution (i.e., scale) for each of the regions.

Wavelet analysis is naturally a multiscale and multiresolution analysis. Mallat and Zhong [11] constructed a dyadic wavelet and the corresponding dyadic wavelet transform (DWT) is equivalent to the Canny edge detection. In wavelet domain, the edge structures present observably at each subband while noise decreases rapidly along the scales. With this observation, Xu et al. [12] proposed a spatially selected filtering technique by multiplying the adjacent DWT scales to enhance the significant structures. Sadler and Swami [13] analyzed the multiscale product of DWT and applied it to the step detection and estimation.

In this paper, we presented a scale-multiplication-based edge detection scheme with the DWT. Two adjacent subbands are multiplied as a product function. Unlike many multiscale edge detectors, where the edge maps were formed at several scales and then synthesized together, our scheme determines edges at the local maxima in the product function after a thresholding. An integrated edge map will be formed efficiently while avoiding the ill-posed edge synthesis process. It will be shown that much improvement is obtained on the localization accuracy and the detection results are better than with either of the two scales only.

2. The dynamic wavelet transform

A function \( \psi(x) \) is called a wavelet if its average is equal to 0. Let \( \zeta(x) = 2^{-2} \zeta(2^{-2}x) \) be the dilation of function \( \zeta(x) \) by dyadic scale \( 2^i \). The DWT of \( f(x) \) at scale \( 2^i \) and position \( x \) is

\[
W_{j} f(x) = f * \psi_{j}(x)
\]  

(1)

Where \( * \) denotes convolution operation. For the details please refer to Daubechies [10] and Mallat [11]. Suppose \( \theta(x) \) is a differentiable smooth function whose integral is 1 and converges to 0 at infinity. Let \( \psi(x) = d\theta(x)/dx \), then

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\[ W_f(x) = f * \psi_f(x) = 2^j \frac{d}{dx}(f * \psi_j)(x) \]  

\( W_f(x) \) is proportional to the first derivative of \( f(x) \) smoothed by \( \psi_j(x) \). The wavelet used in this paper is the Mallat wavelet [11], whose \( \psi_j(x) \) is a cubic spline that approximates the Gaussian function closely. And then the DWT is equivalent to the Canny edge detection.

In 2-D case, two wavelets should be utilized:

\[ \psi'(x,y) = \frac{\partial \psi_j(x,y)}{\partial x}, \quad \psi^2(x,y) = \frac{\partial \psi_j(x,y)}{\partial y} \]  

Denote \( \zeta_j(x,y) = 2^{-j/2} \zeta_j(2^{-j} x, 2^{-j} y) \) the dilation of \( \zeta_j(x,y) \) by \( 2^j \), the wavelet transform of \( f(x,y) \) at scale \( 2^j \) and position \( (x,y) \) has two components

\[ W_f^1(x,y) = f * \psi_1^1(x,y), \quad W_f^2(x,y) = f * \psi_1^2(x,y) \]  

3. The algorithm

3.1. Scale multiplication

(a) **Blocks** \( g \) and its noisy version \( f \).
(b) The DWT of \( g \).
(c) The DWT of noisy \( f \).
(d) The product function \( P_j^f \) with \( j = 1 \sim 3 \).

For signal structures, the DWT amplitudes would increase or keep invariant when increasing the scale \( 2^j \). On the contrary, those of noise will decay rapidly along the scales. Directly multiplying the DWT at two adjacent scales will amplify the edge structures and dilute the noise. The scale product function of \( f(x) \) is defined as the correlation of the DWT of \( f(x) \) at two adjacent scales

\[ P_j^f(x) = W_j f(x) \cdot W_j g(x) \]  

In Fig. 1 (a), a block signal \( g \) and its noisy version \( f \) are illustrated. Their DWT are given in Fig. 1 (b) and (c). At the finest scale the wavelet coefficients are almost dominated by noise. It can also be seen that at small scales the step edges are better localized but some noise may be falsely considered as edges. At the large scales, edges can be detected more correctly but with the decreasing of the accuracy of the edge location. In Fig. 1 (d), the product \( P_j^f, j = 1 \sim 3 \), are illustrated. Apparently the step edges are more observable in \( P_j^f \) than in \( W_f \).

3.2. Thresholding

We assert the edges as the local maxima in \( P_j^f \). A significant edge at \( x_0 \) will occur on both the adjacent scales with the same sign, so that \( P_j^f(x_0) \) should be non-negative. \( P_j^f \) should be thresholded to filter noise.

Let noise \( e \sim N(0, \sigma^2) \) and \( W_f e(x) = e \ast \psi_f(x) \). For expression convenience, we denote \( X_j(x) = W_j e(x) \) and \( Y_j(x) = P_j^f(x) - X_j(x) \cdot X_j(x) \cdot X_j(x) \sim \mathcal{N}(0, \sigma^2) \), where \( \sigma_j = \| \psi_j \| \sigma \) and \( \| \psi_j \| = \sqrt{\int \psi_j^2(x) dx} \).

Denote \( t_u(j) \) the threshold applied to \( Y_j(x) \). It is expected that \( t_u(j) \) could suppress most of the noise, i.e., \( P(Y_j < t_u(j)) \rightarrow 1 \). Normalize \( X_j \) and \( X_j \) as:

\[ \overline{X}_j = X_j / \sigma_j, \quad \overline{X}_j = X_j / \sigma_j \]  

Define \( \overline{Y}_j(x) = \overline{X}_j(x) \cdot \overline{X}_j(x) \) and then \( Y_j(x) = \| \psi_j \| \| \psi_j \| \sigma_j^2 \cdot \overline{Y}_j(x) \) (7)

Let

\[ \overline{Y}_{j,1}(x) = (\overline{X}_j(x) + \overline{X}_j(x)) / 2 \quad \text{and} \quad \overline{Y}_{j,-1}(x) = (\overline{X}_j(x) - \overline{X}_j(x)) / 2 \]  

Thus

\[ \overline{Y}_j(x) = \overline{Y}_{j,1}(x) - \overline{Y}_{j,-1}(x) \]  

and \( \overline{Y}_{j,1} \sim \mathcal{N}(0, \sigma_{j,1}^2) \) and \( \overline{Y}_{j,-1} \sim \mathcal{N}(0, \sigma_{j,-1}^2) \) with

\[ \sigma_{j,1} = \sqrt{\frac{\int \| \psi_j(x) \| \| \psi_j(x) \| \| \psi_j(x) \| \| \psi_j(x) \| dx}{\int \| \psi_j(x) \| \| \psi_j(x) \| \| \psi_j(x) \| \| \psi_j(x) \| dx}} } \]  

\[ \sigma_{j,-1} = \sqrt{\frac{\int \| \psi_j(x) \| \| \psi_j(x) \| \| \psi_j(x) \| \| \psi_j(x) \| dx}{\int \| \psi_j(x) \| \| \psi_j(x) \| \| \psi_j(x) \| \| \psi_j(x) \| dx}} } \]  

(10)

Since there is a strong correlation between \( \psi_j(x) \) and \( \psi_j(x) \), so \( \sigma_{j,1} \) is much larger than \( \sigma_{j,-1} \). Let \( t_u(j) = t_u(j) / \| \psi_j \| \| \psi_j \| \sigma_j^2 \) we have

\[ P(Y_j < t_u(j)) \geq P(Y_j^2 < t_u(j)) = P(Y_j \leq \sqrt{t_u(j)}) \]  

Setting \( \sqrt{t_u(j)} \geq 4 \sigma_{j,1} \) will lead to

\[ P(Y_j < t_u(j)) \leq \sqrt{t_u(j)} / \sqrt{t_u(j)} \geq 4 \sigma_{j,1} \rightarrow 0.9999 \]  

and then

\[ P(Y_j < t_u(j)) \leq \sqrt{t_u(j)} / \sqrt{t_u(j)} \geq 4 \sigma_{j,1} \rightarrow 1 \]  

In real applications, the input is \( f = g + e \) where \( g \) is the original image and then \( W_f = W_g + W_e \). At fine
scales, \( W_j \) will be predominant in \( W_j f \) except for some significant edge structures to be detected. Since the contrast of image singularities and noise is greatly amplified in \( P_j \), threshold \( t_a(j) \) will be much effective in discriminating edges from noise. In our experiments a setting of \( t_a(j) = c \cdot \| \psi_j \| \| \psi_{j+1} \| \sigma^j \sigma_j \), with \( c = 20 \), yields impressive results.

3.3. Two dimensions

In two dimensions, two correlation functions should be defined in \( x \) and \( y \) directions.

\[
P_j^{i-1}(x,y) = W_j f(x,y) - W_j f(x,y)
\]

\[
P_j^{i+1}(x,y) = W_j f(x,y) - W_j f(x,y)
\]

(11)

Similar to 1-D case, for an edge point \((x_0, y_0)\), both \( P_j^{i-1}(x_0, y_0) \) and \( P_j^{i+1}(x_0, y_0) \) should be non-negative but the orientation information of the gradient is lost, which could be recovered from \( W_j f(x_0, y_0) \) and \( W_j f(x_0, y_0) \). The modulus and angle of point \((x, y)\) are defined as

\[
M_j f(x,y) = \sqrt{P_j^{i-1}(x,y)^2 + P_j^{i+1}(x,y)^2}
\]

(12)

\[
A_j f(x,y) = \text{arctan} \left( \frac{\text{sgn}(W_j f(x,y)) \cdot P_j^{i+1}(x,y)}{\text{sgn}(W_j f(x,y)) \cdot P_j^{i-1}(x,y)} \right)
\]

(13)

An edge point is asserted wherever \( M_j f(x,y) \) has a local maximum in the direction of the gradient given by \( A_j f(x,y) \). Similar to section 3.2, let \( t_a^i(j), i = 1, 2, \ldots \), is

\[
t_a^i(j) = c \cdot \| \psi_j \| \| \psi_{j+1} \| \sigma^j \sigma_j
\]

(14)

Where \( c \) is a constant and

\[
\| \psi_j \| = \int \| \psi_j(x,y) \|^2 dxdy
\]

(15)

\[
\sigma^j = \frac{1}{2} \int \left[ \| \psi_j(x,y) \| \psi_{j+1}(x,y) \| \psi_{j+1}(x,y) \| \psi_{j-1}(x,y) \|^2 dxdy
\]

(16)

\( c \) can be chosen around 20. By experimental experience, setting the threshold applied to \( M_j f(x,y) \) as

\[
t_a(j) = 0.8 \cdot \sqrt{t_a^1(j) + t_a^2(j)}
\]

(17)

could achieve satisfying results.

4. Performance analysis

Fig. 2 (a) is a 256x256 isolated noisy step edge. We find edges first at two adjacent scales by Canny edge detection and then in the product of the two scales with our scheme. The figure of merit \( F \) of Pratt [1] is used to evaluate the performance

\[
F = \frac{1}{\max(N_f, N_a)} \sum_{i=1}^{N_f} \frac{1}{1 + \alpha d^2(k)}
\]

(18)

Where \( N_f \) is the number of the actual edges and \( N_a \) is the number of the detected edges. \( d(k) \) denotes the distance from the \( k \)th actual edge to the corresponding detected edge. \( \alpha \) is a scaling constant set to 1/9.

Fig. 2 (c) and (d) are the edge maps by small scale \( 2^1 \) and large scale \( 2^2 \). Fig. 2 (b) is the edge map by our scheme. Denote by \( F_p \) the figure of merit value of Fig. 2 (b) and \( F_1, F_2 \) those of (c) and (d). These values are shown in Table I. As expected, \( F_p \) is the greatest, which means the best performance. \( F_1 \) is less than \( F_2 \) because some false edges are caused by noise.

Next we focus on the localization accuracy of the three edge maps. If the distance \( d(k) \) is not greater than 4 pixels, this edge is considered as a true edge. Denote by \( N \) the total number of true edges that are detected, we define the mean square distance as

\[
D = \frac{1}{N} \sum_{i=1}^{N} d^2(k)
\]

(19)

Table I. The figure of merit values of the two scales and their multiplication.

<table>
<thead>
<tr>
<th></th>
<th>( F_p )</th>
<th>( F_1 )</th>
<th>( F_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9929</td>
<td>0.9496</td>
<td>0.9877</td>
</tr>
</tbody>
</table>

Table II. The mean square distance values of the two scales and their multiplication.

<table>
<thead>
<tr>
<th></th>
<th>( D_p )</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1782</td>
<td>0.2271</td>
<td>0.2887</td>
</tr>
</tbody>
</table>

The smaller the \( D \), the better the localization accuracy will be achieved. Denote by \( D_p \) the mean
square distance of Fig. 2 (b) and $D_1$, $D_2$, those of (c)
and (d). It can be seen from Table II that not only $D_p$ is
less than $D_1$ but also it is less than $D_1$. Scale
multiplication improves the localization accuracy
significantly while keeping high detection efficiency.

![Fig. 4. (a) Noisy Lenna (SNR=16.34dB). (b) Edge
map by scale multiplication scheme.](image)

![Fig. 5. By Canny. (a) $\sigma_s = 0.8$. (b) $\sigma_s = 1.6$.](image)

![Fig. 6. By LOG. (a) $\sigma_s = 1.6$. (b) $\sigma_s = 2.4$.](image)

Next the Lenna image is used to validate the proposed
scheme. The Canny edge detection and LOG algorithms
are employed for comparison. In the two methods,
the standard deviation of the Gaussian function, $\sigma_s$,
is used to adjust the width of the detection filter. In the proposed
scale multiplication based scheme by DWT, we take the
small scale as $2^1$ and then the large scale is $2^3$.

Fig. 4 (a) is the 256x256 noisy Lenna
(SNR=16.34dB). Fig. 4 (b) shows the edge map by the
scale multiplication scheme. Fig. 5 (a)-(b) show the edge
maps generated by Canny edge detection with $\sigma_s = 0.8, 1.6$ respectively. Fig. 6 (a)-(b) are the results by
LOG with $\sigma_s = 1.6, 2.4$. From Fig. 5 and Fig. 6 it can be
seen that when scale is small, finer edges are detected as
well as many false edges. If the scale is enlarged, noise are
suppressed but some edges are also missed or dislocated
(such as the face and hair of Lenna). In Fig. 4 (b) much
better result is achieved. On the one hand, a rich class of
dges, even some fine ones missed by the other two
schemes at fine scale, are detected with better localization
through the scale multiplication. On the other hand, the
dge map is “clear”, which means that false edges are
suppressed well. The experiments on other benchmark
images report the similar results.

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