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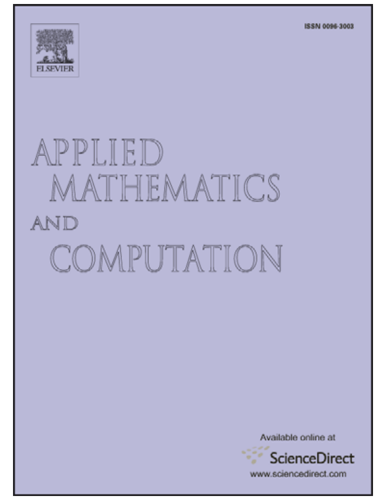

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Estimation of Systems with Statistically-Constrained Inputs

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Abstract -- This paper discusses the estimation of a class of discrete-time linear stochastic systems with statistically constrained unknown inputs (UI), which can represent an arbitrary combination of a class of un-modeled dynamics, random UI with unknown covariance matrix and deterministic UI. In filter design, an upper bound filter is explored to compute, recursively and adaptively, the upper-bounds of covariance matrices of the state prediction error, innovation and state estimate error. Furthermore, the minimum upper bound filter (MUBF) is obtained via online scalar parameter convex optimization in pursuit of the minimum upper-bounds. Two examples, a system with multiple piecewise UIs and a continuous stirred tank reactor (CSTR), are used to illustrate the proposed MUBF scheme and verify its performance.

Keywords:

Adaptive filter, Kalman filtering, disturbance input, stochastic systems, minimum upper bound filter

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I. INTRODUCTION

As optimal linear minimum mean squared-error estimators, Kalman filters are widely used in signal processing and optimal control. The Kalman Filter (KF) considers a linear nominal model with known system parameters and noise statistics. However, the performance of KF will be significantly deteriorated if there exist unknown inputs (UI) to the nominal system, which has motivated the filter design in the presence of disturbance inputs to the system model of the KF. In general, the filter design can be grouped into four categories according to the assumptions on UI.

In the first category, the UI is modeled as stochastic noise with unknown covariance, which is estimated adaptively by computing the correlation of measurements within a time interval [9][10][11]. As pointed out in [9], however, such noise estimation methods are only suitable for a class of stationary time-invariant systems. An adaptive filter [12] was proposed for a class of linear systems that are subject to process disturbances and structured measurement noises with unknown time-variant covariance matrices. The method was further extended to a class of jump Markov nonlinear stochastic systems [13]. The second category considers the UI to be deterministic and piecewise constant [6] for maneuvering target tracking. In a decoupled tracker, one-dimensional target acceleration, as the UI, is estimated using a set of measurements within a moving detection window. In [7], this input estimation method was extended to deal with generalized unknown deterministic inputs, which are represented as a linear weighted combination of several basis functions. The weights are unknown but are assumed to be constant in each moving detection window. This category is specially designed for maneuvering target tracking. In the third category, the UIs are arbitrary but their distribution matrix is known and the matrix rank is less than the rank of measurement matrix [8]. With substitution elimination of the redundant measurements, a disturbance decoupling observer can be designed to obtain the minimum-variance residual for reliable fault diagnosis in the presence of UIs. In the last category, the UI are energy-bounded and the parameter variation is norm-bounded or convex-bounded. The filters, based on offline convex parameter optimization, are designed to purse the best result in the worst-case [1]-[5].

The filters introduced above, based on quite different ideas, address different types of UIs and some filters are application-specific. The actual applications, however, encounter much more complicated UIs. It is highly demanded and significant to design adaptive filters for more generalized UI. To the best of the authors' knowledge, however, little research has been reported on this topic.

In this paper, a statistically constrained UI is considered and defined. The defined UI can represent a linear weighted sum of a class of UIs with dynamic properties, random UIs with unknown covariance, and unknown deterministic UIs. An adaptive filter is proposed via constructing the upper bound filter and pursuing its minimum upper bounds of covariance matrices of the state prediction error, residual and estimation error.

The rest of the paper is organized as follows. Section II formulates the problem, where a class of linear stochastic systems with the statistically-constrained UI is introduced. Section III defines the upper bound filters (UBF) based on the structure of fading Kalman filters, and derives the sufficient conditions of the existence of UBF. Section IV determines the optimal filter parameters in pursuit of the minimum upper bound filter (MUBF). Section V focuses on the online optimization implementation of filter parameters. Section VI performs experiments to illustrate the proposed MUBF and Section VII concludes the paper.

Throughout the paper, for any two real valued matrices with proper dimension, A and B, "$A \geq B$" and "$A > B$" represent that $A - B$ is positive semidefinite and positive definite, respectively. "$0_{n\times p}$" and "I_n" represent the n-by-p zero matrix and the n-by-n unity matrix, respectively. The operator $E\{\bullet\}$ represents the mathematical expectation over the joint distribution of the related random vector. The superscripts "T" and "-1" represent the transpose and inverse of a matrix, respectively.

II. STOCHATIC SYSTEMS WITH GENERALIZED UNKNOWN DISTURBANCE INPUTS

In the standard KF, the discrete-time linear stochastic system model is

$$\begin{cases} x_{k+1} = F_k x_k + B_k u_k + \Gamma_k q_k \\ y_{k+1} = H_{k+1} x_{k+1} + v_{k+1} \end{cases} \tag{1}$$

where $x \in R^n$, $u \in R^l$ and $y \in R^m$ represent the system state, known input and measurement respectively. The system matrix F, input matrix B, noise matrix Γ and measurement matrix H are known. The process noise $q_k \in R^p$ and measurement noise $v_{k+1} \in R^m$ are zero-mean white noises with known covariance $Q_k \geq 0_{p\times p}$ and $R_{k+1} > 0_{m\times m}$, respectively. q, v and the initial state x_0 are assumed to be independent.

In actual applications, however, the modeling errors caused by parameter variations, inaccurate parameter identification and unknown external disturbances may not be well represented by zero-mean process noises with *apriori* known covariance. In such cases the actual system will have significant deviation from the nominal model in (1). Consequently, the KF performance will be deteriorated, or even worse, the estimation errors may be divergent.

Meanwhile, in many engineering applications, there exist various UI to the nominal model. For example, in maneuvering target tracking, the unknown acceleration can be represented as the UI to the constant-velocity movement model; in chemical processes, there widely exist unknown and time-varying time delays, which can be represented by the UI to the model with a nominal time delay. Such situations motivate many researchers to study UI modeling and the corresponding filter design.

We extend (1) to the following time-varying stochastic system

$$\begin{cases} x_{k+1} = F_k x_k + B_k u_k + \Gamma_k q_k + \delta_k \\ y_{k+1} = H_{k+1} x_{k+1} + v_{k+1} \end{cases} \tag{2}$$

where δ_k is independent of $q(j)$ and $v(j+1)$ $(\forall\, j > k \ge 0)$. The introduced term δ_k represents a class of statistically-constrained UIs. Obviously, the traditional system model considered by the KF is a special case of the proposed model (2) with $\delta \equiv 0_{n\times 1}$. It will be shown in the following theorem that δ_k can represent a general class of UIs.

Denote by $\bar{\boldsymbol{X}}^k \triangleq \mathrm{col}\{x_0,\cdots,x_k\}$, $\bar{\boldsymbol{Q}}^{k-1} \triangleq \mathrm{col}\{q_0,\cdots,q_{k-1}\}$, $\bar{\boldsymbol{\delta}}^{k-1} \triangleq \mathrm{col}\{\delta_0,\cdots,\delta_{k-1}\}$, $\bar{\boldsymbol{V}}^k \triangleq \mathrm{col}\{v_1,\cdots,v_k\}$ and $\bar{\boldsymbol{U}}^{k-1} \triangleq \mathrm{col}\{u_0,\cdots,u_{k-1}\}$. We have the following theorem.

Theorem 2.1 Consider the stochastic system (2), the term δ_k can represent the linear combination of multiple UIs as follows

$$\delta_k = \begin{cases} A_k^{(1)} f_k^{(1)}(\bar{\boldsymbol{X}}^k, \bar{\boldsymbol{Q}}^{k-1}, \bar{\boldsymbol{V}}^k, \bar{\boldsymbol{\delta}}^{k-1}, \bar{\boldsymbol{U}}^{k-1}) + A_k^{(2)} f_k^{(2)} + A_k^{(3)} \omega_k & k \ge 1 \\ A_k^{(1)} f_k^{(3)}(\bar{\boldsymbol{X}}^k) + A_k^{(2)} f_k^{(2)} + A_k^{(3)} \omega_k & k = 0 \end{cases} \tag{3}$$

where $A_k^{(1)}$, $A_k^{(2)}$ and $A_k^{(3)}$ are arbitrary deterministic weight matrices with proper dimension; $f_k^{(1)}$, $f_k^{(2)}$ and $f_k^{(3)}$ are arbitrary functions. The noise sequence $\{\omega_k\}$ are assumed to be independent of $\{q_k\}$

and $\{v_k\}$. ■

Proof: See Appendix A. ■

Remark 2.1 As linear or nonlinear functions of $\bar{\boldsymbol{X}}^k$, $\bar{\boldsymbol{Q}}^{k-1}$, $\bar{\boldsymbol{V}}^k$, $\bar{\boldsymbol{\delta}}^{k-1}$ or $\bar{U}^{k-1}$, the functions $f_k^{(1)}$ and $f_k^{(3)}$ represent the UIs with dynamic properties reflecting the effect of model linearization, inaccurate parameter identification and parameter variations on the nominal linear model. The deterministic UI $f_k^{(2)}$ represents the unknown additional deterministic input. The noise ω represents the additional random UI with unknown covariance. As the linear weighted sum of those UIs with arbitrary and even unknown weights, δ_k is a generalized UI and thus system model (2) is able to represent more complicated systems.

III. UPPER BOUND FILTER DESIGN

Definition 3.1 (*Upper Bound Filter*): Consider the following linear filter structure

$$\hat{x}_{k+1|k} = F_k \hat{x}_{k|k} + B_k u_k \tag{4}$$

$$\gamma_{k+1} = y_{k+1} - H_{k+1}\hat{x}_{k+1|k} \tag{5}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1}\gamma_{k+1} \tag{6}$$

$$P^*_{k+1|k} = \alpha_k F_k P^*_{k|k} F_k^T + \Gamma_k Q_k \Gamma_k^T \tag{7}$$

$$V^*_{k+1} = H_{k+1} P^*_{k+1|k} H^T_{k+1} + R_{k+1} \tag{8}$$

$$P^*_{k+1|k+1} = (I - K_{k+1}H_{k+1})P^*_{k+1|k}(I - K_{k+1}H_{k+1})^T + K_{k+1}R_{k+1}K^T_{k+1} \tag{9}$$

where $\hat{x}_{k+1|k}$, γ_{k+1} and $\hat{x}_{k+1|k+1}$ are the state prediction, residual and estimate at time k+1, respectively. The filter (4)-(9) for system (2) is said to be the Upper Bound Filter (UBF) if there exist a fading factor $\alpha_k \geq 1$ and filter gain K_{k+1} satisfying

(i) $$P^*_{k+1|k} \geq P_{k+1|k} \triangleq E\{\tilde{x}_{k+1|k}\tilde{x}^T_{k+1|k}\} \tag{10}$$

(ii) $$V^*_{k+1} \geq V_{k+1} \triangleq E\{\gamma_{k+1}\gamma^T_{k+1}\} \tag{11}$$

(iii) $$P^*_{k+1|k+1} \geq P_{k+1|k+1} \triangleq E\left\{\tilde{x}_{k+1|k+1}\tilde{x}^T_{k+1|k+1}\right\} \tag{12}$$

where $\tilde{x}_{k|k} \triangleq x_k - \hat{x}_{k|k}$, $\tilde{x}_{k+1|k} \triangleq x_{k+1} - \hat{x}_{k+1|k}$ and the filter gain K_{k+1} is a function of $P^*_{k+1|k}$ and V^*_{k+1}. ■

Remark 3.1 In Definition 3.1, the UI is unknown and thus state prediction has to be based on the nominal system (1). Similar to the fading Kalman filter (FKF) [14] , the effect of the UI on state prediction is expected to be compensated via the fading factor in (7). In fact, the UBF and FKF are different. The fading factor in the FKF is introduced to rescale covariance matrix of state prediction error derived from the nominal system. However, it is not theoretically strict to reconstruct an $n \times n$ square matrix only from a scalar. Different from FKF, the proposed UBF is to construct the upper bounds of covariance matrices of the state prediction error, innovation and state estimate error via the fading factor.

Before designing the UBF, we need to know whether a UBF exists. The following Theorem 3.1 presents the sufficient conditions of the existence of the UBF.

Theorem 3.1 There exist a UBF (4)-(12) for system (2) and both $P^*_{k+1|k}$ and $P^*_{k+1|k+1}$ are positive definite, if the following conditions hold:

(i) $P^*_{0|0}$ is positive definite and $P^*_{0|0} - P_{0|0}$ is positive semi-definite

(ii) $F_k \ \forall k$ is of full rank. ■

Proof: See Appendix B. ■

In the traditional KF, $P_{0|0}$ is known *apriori*. In the UBF proposed here, only its upper bound $P^*_{0|0} > 0$ is needed. The first condition of Theorem 3.1 is easy to be satisfied in filter design. The second condition of Theorem 3.1 presents the constraint on the system (2) and is satisfied in many applications. For example, if model (2) is derived from the discretization of a continuous-time system, then this condition will be always satisfied.

IV. MINIMUM UPPER BOUNDS

In the previous section, Theorem 3.1 explores the existence of a UBF. An interesting question is whether there exist the minimum upper bounds and how to design the filter parameters in order to obtain the best filtering accuracy in the worst case. In this section, we show that the minimum upper bounds exist, and further present the corresponding filter parameters.

Define the following set $\Lambda_k = \left\{ \alpha_k \mid \alpha_k \geq 1, P^*_{k+1|k} \geq P_{k+1|k}, V^*_{k+1} \geq V_{k+1},\ P^*_{k+1|k+1} \geq P_{k+1|k+1} \right\}$. According to Definition 3.1, the set Λ_k will not be empty if a UBF exists. We have the following theorems.

Theorem 4.1 Given any positive definite symmetric matrix $P^*_{k|k}$ in a UBF (4)-(12) for system (2), there exists a unique minimum fading factor α_k^{Opt} in Λ_k to achieve the minimum upper bound of covariance matrix of the state prediction error, i.e.

$$\alpha_k^{Opt} = \min\left\{ \alpha_k \in \Lambda_k \right\} \tag{13}$$

satisfying

$$P_{k+1|k} \leq P^*_{k+1|k}\Big|_{\alpha_k^{Opt}} \leq P^*_{k+1|k}\Big|_{\alpha_k} \tag{14}$$

The fading factor α_k^{Opt} also results in the minimum upper bounds of covariance matrices of filter residual and state estimate error, given any filter gain, i.e.

$$V_{k+1} \leq V^*_{k+1}\Big|_{\alpha_k^{Opt}} \leq V^*_{k+1}\Big|_{\alpha_k} \tag{15}$$

$$P(k+1 \mid k+1) \leq P^*_{k+1|k+1}\Big|_{\alpha_k^{Opt}} \leq P^*_{k+1|k+1}\Big|_{\alpha_k} \tag{16} \quad \blacksquare$$

Proof: See Appendix C. ■

Theorem 4.2 Given any positive definite symmetric matrix $P^*_{k+1|k}$ in a UBF (4)-(12) for system (2), there exists an optimal filter gain to obtain the minimum upper bound of the covariance matrix of state estimate error, i.e.

$$K_{k+1}^{Opt} = P^*_{k+1|k} H^T_{k+1} \left(V^*_{k+1} \right)^{-1} \tag{17}$$

satisfying

$$P_{k+1|k+1} \leq P^*_{k+1|k+1}\Big|_{K_{k+1}^{Opt}} \leq P^*_{k+1|k+1}\Big|_{K_{k+1}} \tag{18}$$

where

$$P^*_{k+1|k+1}\Big|_{K^{opt}_{k+1}} = \left(\left(P^*_{k+1|k}\right)^{-1} + H^T_{k+1}R^{-1}_{k+1}H_{k+1}\right)^{-1} \tag{19}$$ ■

Proof: See Appendix D. ■

With Theorems 4.1 and 4.2, we know that there exists a unique Minimum Upper Bound Filter (MUBF), reaching the minimum upper bounds among all UBFs. The MUBF has optimal parameters, i.e. α_k^{Opt} and K_{k+1}^{Opt}. Next, we focus on how to determine α_k^{Opt} and thus propose the implementation of the MUBF.

V. MINIMUM UPPER BOUND FILTER DESIGN

Theorem 5.1 The following scalar convex optimization has one and only one solution, which is equal to the minimum fading factor

$$\alpha_k^{Opt} = \min \alpha_k \tag{20}$$

subject to

$$\text{(i)}\quad H_{k+1}\left(\alpha_k F_k P^*_{k|k} F_k^T + \Gamma_k Q_k \Gamma_k^T\right) H^T_{k+1} \geq V_{k+1} - R_{k+1} \tag{21}$$

$$\text{(ii)}\quad \alpha_k \geq 1$$

if the following four conditions are satisfied

(i) $P^*_{0|0}$ is positive definite and $P^*_{0|0} - P_{0|0}$ is positive semi-definite;

(ii) F_k is of full rank;

(iii) H_k is of full row rank, i.e. $rank\{H_k\} = m$, $\forall k \geq 1$;

(iv) the filter gain is chosen to its optimal value K_{k+1}^{Opt} in (17). ■

Proof: See Appendix E. ■

The MUBF algorithm can be summarized as follows.

Step 1: Let k=0. Initialize $\hat{x}_{0|0}$ and $P^*_{0|0}$.

Step 2: Compute the state prediction $\hat{x}_{k+1|k}$ by (4).

Step 3: After obtain the measurement at time k+1, compute γ_{k+1} by (5).

Step 4: Run the optimization of (20) to determine α_k^{Opt}, where V_{k+1} is substituted by its estimate $\hat{V}_{k+1} = \gamma_{k+1}\gamma_{k+1}^T$. The convex constrained optimization can be solved via linear matrix inequalities (LMI) [15].

Step 5: Compute $P_{k+1|k}^*$ by (7), V_{k+1}^* by (8) and K_{k+1}^{Opt} by (18).

Step 6: Determine $\hat{x}_{k+1|k+1}$ and $P_{k+1|k+1}^*$ by (6) and (9), respectively.

Step 7: Let k=k+1 and go to Step 2.

VI. EXPERIMENTAL RESULTS

In this section, two examples are used to illustrate the proposed MUBF. In the first example, a system with multiple types of UIs is simulated to test the performance of MUBF; while in the second example, a real system of the continuous stirred tank reactor (CSTR) is used to validate the efficiency of MUBF.

A) A simulated system with different UIs

Consider the system (2) with $F = \begin{bmatrix} 0.8 & 0.3 \\ -0.3 & 0.9 \end{bmatrix}$, $R = \begin{bmatrix} 400 & 0 \\ 0 & 400 \end{bmatrix}$, $Bu \equiv 0_{2\times 1}$, $\Gamma = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $H = I_{2\times 2}$ and $Q = 5$. The UI δ_k is considered to be 0, time-invariant UI, periodic UI, random UI, and time-delayed UI in piece-wise style as follows

$$\delta_k = \begin{cases} [0 \;\; 0]^T & 1 \le k \le 50 \\ [0 \;\; 30\sin(0.5k-25)]^T & 51 \le k \le 100 \\ [0 \;\; -20]^T & 101 \le k \le 150 \\ [\varepsilon_k^{(1)} \;\; \varepsilon_k^{(2)}]^T & 151 \le k \le 200 \\ [[0 \;\; -1]x_{k-1}\beta_k \;\; 0]^T & 201 \le k \le 250 \end{cases}$$

where $\varepsilon_k^{(i)}$ $(\forall k, i=1,2)$ is Gaussian white noise with variance of 80, β_k is a square wave with peak magnitude of ±0.3 and the half time period of 5. The first and second state components to be estimated are shown in Fig.1.

For the above time-invariant system configuration without δ_k, the first category of filters introduced in the Introduction Section can be designed. Here we employ the adaptive suboptimal filter (ASF) [10] for comparison with the proposed filter. The other three categories of filters introduced in the Introduction Section can not be applied because their design conditions are not satisfied. As shown in Fig. 2, the ASF, which treats the UI as stationary random noise, works much worse than the proposed MUBF, especially in the presence of deterministic UIs. It means that not all UIs can be properly treated as stationary random noise and the ASF is not suitable to deal with the general UI.

The fading factor is determined *a priori* in FKF [14], while it is adaptively optimized in MUBF. We implemented the FKFs with the fading factor ranging from 1 to ∞ for comparison. The FKF with α=1 is equivalent to the traditional KF while the FKF with α=∞ is equivalent to the Least Square Estimation. As shown in Figs.3-5, the FKFs with different fading factors cannot result in consistent filtering accuracy in different UIs. Table 1 lists the Root Mean Square Error (RMSE) of the filtering outputs of different schemes in each piece-wise UI. It is shown that the proposed MUBF, as one adaptive filter based on online parameter optimization, is robust to different UIs.

B) A time delayed stochastic system

This experiment is on the joint estimation of feed flow rate and time delay in the continuous stirred tank reactor (CSTR), where the evolution of feed flow rate and time delay is hard to model and hence we model their changes as the UI. Time delay is a special structure parameter that exists in many systems. As pointed out in [16], the closed-loop system may be unstable if the modeled time delay does not coincide with the unknown and/or time-varying time delay. Here we consider the CSTR described by Henson *et al* [17]:

$$\begin{cases} \dot{C}a(t) = \dfrac{\tau(t)}{U}(C_{Af} - Ca(t)) - k_0 \exp(-\dfrac{E}{DT(t)})Ca(t) \\ \dot{T}(t) = \dfrac{\tau(t)}{U}(\overline{t}^{\,f} - T(t)) + \dfrac{-\Delta H}{\overline{\rho} C_p} k_0 \exp(-\dfrac{E}{DT(t)})Ca(t) + \dfrac{Z}{U\overline{\rho}C_p}(\overline{t}^{\,c}(t-\phi(t)) - T(t)) \end{cases} \tag{22}$$

where Ca is the reactor concentration, T is the reactor temperature, $\overline{t_c}$ is the coolant temperature, τ is the feed flow rate, C_{Af} is the feed concentration and ϕ is the time delay.

In chemical engineering, the reactor temperature, T, can be obtained accurately. Consider the case that the reactor concentration is constant. Via the Eular discretization with sampling interval dt, we have

$$
\begin{aligned}
T_{k+1} &= T_k + dt\frac{\tau_k}{U}(\bar{t}^f - T_k) + dt\frac{-\Delta H}{\bar{\rho} C_p} k_0 \exp(-\frac{E}{DT_k}) Ca_k \\
&+ dt\frac{Z}{U\bar{\rho} C_p}\bar{t}^c_{k+1-\phi_{k+1}-T_k} + e_{k+1}
\end{aligned}
\tag{23}
$$

where e, the discretization error, is a zero-mean white noise with covariance R_{k+1}. The feed flow rate, τ, and the time delay, ϕ, are time varying and change randomly. It is required to estimate them jointly. Let the state be $x = [\tau \quad \phi]^T$, we model the system as (2) with the following parameters:

$$F = I_2, Bu = 0_{2\times 1}, \ \Gamma q = 0_{2\times 1}, \ \delta_k = x_{k+1} - x_k, \ v = e$$

$$y_{k+1} = T_{k+1} - T_k + dt\frac{\tau_k}{U}T_k + dt\frac{\Delta H}{\bar{\rho} C_p} k_0 \ Ca_k \exp(-\frac{E}{DT_k}) + \frac{dtZ}{U\bar{\rho} C_p}T_k$$

$$H_{k+1} = [\ (\bar{t}^f - T_k)/U \quad dt\frac{Z}{U\bar{\rho} C_p}(\bar{t}^c_{k-\phi_{k+1}}, -\bar{t}^c_{k+1-\phi_{k+1}})\,|_{\phi_{k+1}=\hat{\phi}_k}\ ]$$

In the above modeling, the unknown changes of feed flow rate and time-delay are modeled as $\delta(k)$ because we do not have any *a priori* information about such changes.

The nominal parameters of the CSTR are Ca =0.1 mol/L, C_{Af} =1 mol/L, $\bar{t}^f$ =350 K, U =100 L, Z =5×10^4 J/(min·K), k_0 =7.2×10^{10}/min, ΔH =–5×10^4 J/mol, *E/D*=8750 K, $\bar{\rho}$ =1000 g/L, and C_p =0.239 J/(g·K). $\bar{t}^c$ is designed to be a square wave with peak magnitude 309.9×(1±0.005)K. The half time period of $\bar{t}^c$ is 1 min so that the time delay less than 2 min is expected to be estimated. The sampling interval dt is 0.2 min and the variance of the discretization error is $R_{k+1} = 0.05^2$. The initial state of the CSTR is $x_0 = [100 \ \ 1]^T$. The actual time-varying parameter τ (in L/min) and time delay ϕ (in 0.2 min) are

$$\tau(k) = \begin{cases} 100 & 0 < k \le 50 \\ 95 + 0.1k & 50 < k \le 150 \\ 120 & 150 < k \le 250 \\ 110 & 250 < k \le 300 \end{cases}, \quad \phi_k = \begin{cases} 1 & 0 < k \le 100 \\ 3 & 100 < k \le 200 \\ 5 & 200 < k \le 300 \end{cases}$$

All the four categories of filters introduced in the Introduction Section can not be applied to the above CSTR system. The first category of filters can not be used because the necessary condition of filter existence in [9] can not be satisfied due to the time-variant measurement matrix. The systems considered in [10] and

[13] are much different from the CSTR system and their methods can not be used either. The second category of filters, which reconstructs inputs via measurements, requires that the dimension of inputs should not be less than that of measurements. However, this condition can not be satisfied because the CSTR system has one-dimensional measurement while two-dimensional input needs to be estimated. The third category is also unsuitable because it assumes that the disturbance should be structured and the disturbance dimension should be less than the measurement dimension. The last category is not applicable because it needs to know the bounds of disturbance and its existence depends on the system parameters. Here, we compare the proposed MUBF with FKF.

The filtering results by using the FKF with α=1 (i.e. the standard EKF, which has been widely utilized for many nonlinear systems, including gene regulatory networks [18]) are shown in Fig. 6. We see that it fails to estimate the time-variant state because of the absence of adaptation to the un-modeled dynamics. In contrast, the proposed MUBF, as shown in Fig.7, presents satisfying filtering results. The results of FKF with different fading factors are illustrated in Figs. 8-10, from which we can see that unsuitable fading factors will lead to unacceptable results. Comparing Fig.7 with Fig.9, we can see that FKF with the best fading factor gets slightly better estimation in feed flow rate τ but much worse estimation in time delay ϕ than the proposed MUBF. It validates the fact that the FKF with *a priori* fading factor is not expected to be suitable for time-variant system with time-variant and complicated UIs. In practice, the CSTR system may switch between multiple operation modes, which are significantly different from each other in nominal parameters and state change rules. In such cases, determining the FKF fading factor via trial and error becomes too complicated to implement.

VII. CONCLUSION

We proposed an adaptive filtering scheme for a class of stochastic systems that are subject to statistically constrained disturbance input, which can describe an arbitrary combination of a class of un-modeled dynamics, random noise with unknown covariance and unknown deterministic inputs. The upper bound filter to the systems was presented. By minimizing the upper bounds, we transformed the design of minimum upper bound filter (MUBF) into an online constrained scalar optimization. Two examples were used to illustrate and validate the proposed MUBF and the results showed that the MUBF can effectively handle different types of UI, which cannot be well processed by conventional filters such as fading Kalman filters.

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APPENDIX A. Proof of Theorem 2.1

For simplicity without confusing in Appendix A, we represent $f_k^{(1)}$, $f_k^{(2)}$ and $f_k^{(3)}$ by $f^{(1)}$, $f^{(2)}$ and $f^{(3)}$, respectively. Similarly, weights $A_k^{(1)}$, $A_k^{(2)}$, $A_k^{(3)}$ are represented by $A^{(1)}$, $A^{(2)}$, $A^{(3)}$, respectively. The noises ω_k is represented by ω.

First let's consider the case of $k=0$. From the statement in (1) that q, v and the initial state x_0 are independent, we have that $f^{(3)}$ is independent of q and v. Further considering that $A^{(1)}$ $A^{(2)}$, $A^{(3)}$ and $f^{(2)}$ are deterministic, ω is uncorrelated with both q and v, and q, v are zero-mean, we have

$$E\left\{\delta_0 q_j^T\right\} = \left(A^{(1)}E\left\{f^{(3)}\right\} + A^{(2)}f^{(2)} + A^{(3)}E\{\omega\}\right)E\left\{q_j^T\right\} = 0_{n\times p}$$

$$E\left\{\delta_0 v_{j+1}^T\right\} = \left(A^{(1)}E\left\{f^{(3)}\right\} + A^{(2)}f^{(2)} + A^{(3)}E\{\omega\}\right)E\left\{v_{j+1}^T\right\} = 0_{n\times m}$$

Suppose δ_i ($i \le k-1$) can be represented by (3). We focus on testifying the statistical constraint on δ_k. From the dynamic model in (2), we can see that $\bar{\boldsymbol{X}}^k$ is a linear function of x_0, $\bar{\boldsymbol{Q}}^{k-1}$, $\bar{\boldsymbol{\delta}}^{k-1}$ and $\bar{U}^{k-1}$. Thus $f^{(1)}$ is the function of x_0, $\bar{\boldsymbol{Q}}^{k-1}$, $\bar{\boldsymbol{V}}^k$, $\bar{\boldsymbol{\delta}}^{k-1}$ and $\bar{U}^{k-1}$. Because q, v and the initial state x_0 are independent, q, v are white, and $\bar{U}^{k-1}$ is deterministic, we have that $f^{(1)}$ is independent of q_j and v_{j+1}. Thus we obtain

$$E\left\{\delta_k q_j^T\right\} = \left(A^{(1)}E\left\{f^{(1)}\right\} + A^{(2)}f^{(2)} + A^{(3)}E\{\omega\}\right)E\left\{q_j^T\right\} = 0_{n\times p}$$

$$E\left\{\delta_k v_{j+1}^T\right\} = \left(A^{(1)}E\left\{f^{(1)}\right\} + A^{(2)}f^{(2)} + A^{(3)}E\{\omega\}\right)E\left\{v_{j+1}^T\right\} = 0_{n\times m} \qquad \blacksquare$$

APPENDIX B. Proof of Theorem 3.1

First, we show that $P^*_{k+1|k}$ and $P^*_{k+1|k+1}$ are positive definite matrices. As in the first condition of Theorem 3.1, $P^*_{0|0}$ is positive definite. Here we assume $P^*_{k|k}$ is positive definite and try to prove that $P^*_{k+1|k}$ and $P^*_{k+1|k+1}$ are positive definite matrices using mathematical induction.

Since Q_k is nonnegative definite and $\alpha_k \geq 1$, we have the following inequality form (7)

$$P^*_{k+1|k} = \alpha_k F_k P^*_{k|k} F_k^T + \Gamma_k Q_k \Gamma_k^T \geq \alpha_k F_k P^*_{k|k} F_k^T \geq F_k P^*_{k|k} F_k^T \tag{B.1}$$

In the fact that $P^*_{k|k}$ is positive definite, and F_k is of full rank (the second condition of Theorem 3.1), we further have $P^*_{k+1|k}$ is positive definite from (B.1).

To any filtering gain K_{k+1}, we define $\Delta K_{k+1} = K_{k+1} - P^*_{k+1|k} H_{k+1}^T (V^*_{k+1})^{-1}$ and derive (9) as follows

$$\begin{aligned} P^*_{k+1|k+1} &= P^*_{k+1|k} - P^*_{k+1|k} H_{k+1}^T (V^*_{k+1})^{-1} H_{k+1} P^*_{k+1|k} + \Delta K_{k+1} V^*_{k+1} \Delta K_{k+1}^T \geq P^*_{k+1|k} - P^*_{k+1|k} H_{k+1}^T (V^*_{k+1})^{-1} H_{k+1} P^*_{k+1|k} \\ &\geq \left[\left(P^*_{k+1|k} \right)^{-1} + H_{k+1}^T R_{k+1}^{-1} H_{k+1} \right]^{-1} \end{aligned} \tag{B.2}$$

Using above conclusion that $P^*_{k+1|k}$ is positive definite, we find that $P^*_{k+1|k+1}$ is positive definite

Then we prove that there must exist the fading factor to guarantee (10)-(12). Put (4)-(6) into the definition of mean square error matrices in (10)-(12), we have

$$P_{k+1|k} = E\left\{ [F_k \tilde{x}_{k|k} + \delta_k][F_k \tilde{x}_{k|k} + \delta_k]^T \right\} + \Gamma_k Q_k \Gamma_k^T \tag{B.3}$$

$$V_{k+1} = H_{k+1} P_{k+1|k} H_{k+1}^T + R_{k+1} \tag{B.4}$$

$$P_{k+1|k+1} = (I - K_{k+1} H_{k+1}) P_{k+1|k} (I - K_{k+1} H_{k+1})^T + K_{k+1} R_{k+1} K_{k+1}^T \tag{B.5}$$

where above derivation utilizes the constraint that δ_k is independent of $q(j)$ and $v(j+1)$ ($\forall\ j > k \geq 0$). Substitute (7) by (B.3), we have

$$P^*_{k+1|k} - P_{k+1|k} = \alpha_k F_k P^*_{k|k} F_k^T - E\left\{ [F_k \tilde{x}_{k|k} + \delta_k][F_k \tilde{x}_{k|k} + \delta_k]^T \right\}$$

From (B.2) and the second condition of Theorem 3.1, we have that $F_k P^*_{k|k} F_k^T$ is positive definite and thus its minimum eigenvalue is positive, i.e. $\sigma_{\min}\left\{F_k P^*_{k|k} F_k^T\right\} > 0$.

The real symmetric matrices can be diagonalized as

$$F_k P^*_{k|k} F_k^T = U_k^{(1)} \Lambda_k^{(1)} \left(U_k^{(1)}\right)^T, \quad E\left\{[F_k \tilde{x}_{k|k} + \delta_k][F_k \tilde{x}_{k|k} + \delta_k]^T\right\} = U_k^{(2)} \Lambda_k^{(2)} \left(U_k^{(2)}\right)^T$$

where $\Lambda_k^{(1)}$ and $\Lambda_k^{(2)}$ are positive semi-definite and diagonal, $\left(U_k^{(1)}\right)^T = \left(U_k^{(1)}\right)^{-1}$, and $\left(U_k^{(2)}\right)^T = \left(U_k^{(2)}\right)^{-1}$. Furthermore we have

$$\alpha_k F_k P^*_{k|k} F_k^T = \alpha_k U_k^{(1)} \Lambda_k^{(1)} \left(U_k^{(1)}\right)^T \geq \alpha_k U_k^{(1)} (\sigma_{\min}(\Lambda_k^{(1)}) I_n) \left(U_k^{(1)}\right)^T = \alpha_k \sigma_{\min}(\Lambda_k^{(1)}) I_n$$

$$U_k^{(2)} \Lambda_k^{(2)} \left(U_k^{(2)}\right)^T \leq U_k^{(2)} (\sigma_{\max}(\Lambda_k^{(2)}) I_n) \left(U_k^{(2)}\right)^T = \sigma_{\max}(\Lambda_k^{(2)}) I_n$$

where $\sigma_{\min}(\cdot)$ and $\sigma_{\max}(\cdot)$ are the minimum and maximum eigenvalues of the corresponding real symmetric matrix, respectively.

Choosing $\alpha_k \geq \dfrac{\sigma_{\max}(\Lambda_k^{(2)})}{\sigma_{\min}(\Lambda_k^{(1)})}$, we obtain

$$P^*_{k+1|k} \geq P_{k+1|k} \tag{10}$$

Subtract (8) by (B.4) and use (10), we have

$$V^*_{k+1} = V_{k+1} + H_{k+1}(P^*_{k+1|k} - P_{k+1|k}) H_{k+1}^T \geq V_{k+1} \tag{11}$$

Subtract (9) by (B.5) and use (10), we have

$$P^*_{k+1|k+1} = P_{k+1|k+1} + (I - K_{k+1} H_{k+1})(P^*_{k+1|k} - P_{k+1|k})(I - K_{k+1} H_{k+1})^T \geq P_{k+1|k+1} \tag{12}$$ ■

APPENDIX C. Proof of Theorem 4.1

To any $\alpha_k \in \Lambda_k$, (13) leads to

$$\alpha_k \geq \alpha_k^{Opt} \tag{C.1}$$

From (7) and (C.1), we have

$$P^*_{k+1|k}\Big|_{\alpha_k^{Opt}} = \alpha_k^{Opt} F_k P^*_{k|k} F_k^T + \Gamma_k Q_k \Gamma_k^T \leq \alpha_k F_k P^*_{k|k} F_k^T + \Gamma_k Q_k \Gamma_k^T = P^*_{k+1|k}\Big|_{\alpha_k} \tag{C.2}$$

Due to $\alpha_k^{Opt} \in \Lambda_k$, we have $P^*_{k+1|k}\Big|_{\alpha(k)=\alpha_k^{Opt}} \geq P_{k+1|k}$. Further with (C.2), we have (14).

From (8) and (C.2), we have

$$V_{k+1}^{*}\Big|_{\alpha_k^{Opt}} = H_{k+1} P_{k+1|k}^{*}\Big|_{\alpha_k^{Opt}} H_{k+1}^{T} + R_{k+1} \le H_{k+1} P_{k+1|k}^{*}\Big|_{\alpha_k} H_{k+1}^{T} + R_{k+1} = V_{k+1}^{*}\Big|_{\alpha_k} \tag{C.3}$$

From (9), we have

$$P_{k+1|k+1}^{*}\Big|_{\alpha_k^{Opt}} = (I - K_{k+1}H_{k+1}) P_{k+1|k}^{*}\Big|_{\alpha_k^{Opt}} (I - K_{k+1}H_{k+1})^{T} + K_{k+1}R_{k+1}K_{k+1}^{T} \tag{C.4}$$

and

$$P_{k+1|k+1}^{*}\Big|_{\alpha_k} = (I - K_{k+1}H_{k+1}) P_{k+1|k}^{*}\Big|_{\alpha_k} (I - K_{k+1}H_{k+1})^{T} + K_{k+1}R_{k+1}K_{k+1}^{T} \tag{C.5}$$

Subtract (C.4) from (C.5) and use (C.2), we have

$$P_{k+1|k+1}^{*}\Big|_{\alpha_k^{Opt}} \le P_{k+1|k+1}^{*}\Big|_{\alpha_k} \tag{C.6}$$

Due to $\alpha_k^{Opt} \in \Lambda_k$, we have $V_{k+1}^{*}\Big|_{\alpha_k^{Opt}} \ge V_{k+1}$ and $P_{k+1|k+1}^{*}\Big|_{\alpha_k^{Opt}} > P_{k+1|k+1}$. Combining them with (C.3)-(C.4), we obtain (15)-(16).

At last, let us testify the uniqueness of the optimal fading factor. As shown in (C.2), we have

$$P_{k+1|k}^{*}\Big|_{\alpha_k^{Opt}} - P_{k+1|k}^{*}\Big|_{\alpha_k} = \left(\alpha_k^{Opt} - \alpha_k\right) F_k P_{k|k}^{*} F_k^{T}$$

Because $P_{k|k}^{*}$ is positive definition as stated in Theorem 4.1, we have

(i) If $P_{k+1|k}^{*}\Big|_{\alpha_k^{Opt}} - P_{k+1|k}^{*}\Big|_{\alpha_k}$ >0, then $\alpha_k^{Opt} > \alpha_k$.

In this case, α_k is not the optimal parameter because α_k^{Opt} obtains the lower upper bound.

(ii) If $P_{k+1|k}^{*}\Big|_{\alpha_k^{Opt}} - P_{k+1|k}^{*}\Big|_{\alpha_k}$ <0, then $\alpha_k^{Opt} < \alpha_k$.

This case is impossible because $\alpha_k^{Opt} = \min\left\{\alpha_k \in \Lambda_k\right\}$ as defined in (13).

(iii) If $P_{k+1|k}^{*}\Big|_{\alpha_k^{Opt}} - P_{k+1|k}^{*}\Big|_{\alpha_k}$ =0, then $\alpha_k^{Opt} = \alpha_k$ or $F_k \equiv 0_{n\times n}$.

The case of $F_k \equiv 0_{n\times n}$ means it fails to model the system dynamics in (2) and thus only measurement information is available for estimation. It is out of the scope of estimation problem of dynamic systems. Therefore it is concluded that α_k^{Opt} is unique. ■

APPENDIX D. Proof of Theorem 4.2

Because $P^{*}_{k+1|k}$ and R_{k+1} are positive definite, $H_{k+1}P^{*}_{k+1|k}H^{T}_{k+1}+R_{k+1}$ is also positive definite and can be expressed by

$$V^{*}_{k+1}=H_{k+1}P^{*}_{k+1|k}H^{T}_{k+1}+R_{k+1}=S_{k+1}S^{T}_{k+1} \tag{D.1}$$

where S_{k+1} is an m-by-m matrix of full rank. Denote

$$M_{k+1}=K_{k+1}S_{k+1}-P^{*}_{k+1|k}H^{T}_{k+1}S^{-1}_{k+1}\Big|_{K^{Opt}_{k+1}}$$

and (9) can be rewritten as

$$P^{*}_{k+1|k+1}=(I-K_{k+1}H_{k+1})P^{*}_{k+1|k}(I-K_{k+1}H_{k+1})^{T}+K_{k+1}R_{k+1}K^{T}_{k+1}=K_{k+1}V^{*}_{k+1}K^{T}_{k+1}-K_{k+1}H_{k+1}P^{*}_{k+1|k}-\left(K_{k+1}H_{k+1}P^{*}_{k+1|k}\right)^{T}+P^{*}_{k+1|k}$$
$$=M_{k+1}M^{T}_{k+1}+P^{*}_{k+1|k}-P^{*}_{k+1|k}H^{T}_{k+1}\left(V^{*}_{k+1}\right)^{-1}H_{k+1}P^{*}_{k+1|k}\geq P^{*}_{k+1|k}-P^{*}_{k+1|k}H^{T}_{k+1}\left(V^{*}_{k+1}\right)^{-1}H_{k+1}P^{*}_{k+1|k}$$

From the above, we can see that $P^{*}_{k+1|k+1}$ reaches its lower bound if and only if $M_{k+1}=0_{n\times m}$. Thus we obtain the optimal filter gain

$$K^{Opt}_{k+1}=P^{*}_{k+1|k}H^{T}_{k+1}\left(S_{k+1}S^{T}_{k+1}\right)^{-1}=P^{*}_{k+1|k}H^{T}_{k+1}\left(V^{*}_{k+1}\right)^{-1} \tag{D.2}$$

satisfying

$$P^{*}_{k+1|k+1}\Big|_{K^{Opt}_{k+1}}\leq P^{*}_{k+1|k+1}\Big|_{K_{k+1}} \tag{D.3}$$

Using the well-known matrix inversion theorem, $P^{*}_{k+1|k+1}\Big|_{K^{Opt}_{k+1}}$ can be further expressed as

$$P^{*}_{k+1|k+1}\Big|_{K^{Opt}_{k+1}}=P^{*}_{k+1|k}-P^{*}_{k+1|k}H^{T}_{k+1}\left(V^{*}_{k+1}\right)^{-1}H_{k+1}P^{*}_{k+1|k}=\left(\left(P^{*}_{k+1|k}\right)^{-1}+H^{T}_{k+1}R^{-1}_{k+1}H_{k+1}\right)^{-1} \tag{D.4}$$

With $P^{*}_{k+1|k}>0_{n\times n}$, (D.4) further leads to $P^{*}_{k+1|k+1}\Big|_{K^{Opt}_{k+1}}>0_{n\times n}$. Thus K^{Opt}_{k+1} satisfies the third condition of Theorem 3.1 and the upper bound can be guaranteed:

$$P_{k+1|k+1}\leq P^{*}_{k+1|k+1}\Big|_{K^{Opt}_{k+1}} \tag{D.5}$$

Combining (D.3) and (D.5), we obtain (18). ■

APPENDIX E. Proof of Theorem 5.1

The first two conditions of Theorem 5.1 are the same as the conditions of Theorem 3.1. Thus a UBF exists and $P^*_{k|k} > 0_{n\times n}$ for all $0 < k \le N$, according to Theorem 3.1.

From the third condition of Theorem 5.1, we have that $H^T_{k+1}H_{k+1}$ is positive definite and thus $\left(H^T_{k+1}H_{k+1}\right)^{-1}$ exists. Substitute (7), (8) and (B.3) into (21), we have

$$V^*_{k+1} = H_{k+1}\left(\alpha_k F_k P^*_{k|k} F^T_k + \Gamma_k Q_k \Gamma^T_k\right) H^T_{k+1} + R_{k+1} \ge V_{k+1} \tag{11}$$

Left-multiply and right-multiply (21) by H^T_{k+1} and H_{k+1} respectively, we have

$$H^T_{k+1}H_{k+1}\left(\alpha_k F_k P^*_{k|k} F^T_k + \Gamma_k Q_k \Gamma^T_k\right) H^T_{k+1}H_{k+1} \ge H^T_{k+1}\left(V_{k+1} - R_{k+1}\right) H_{k+1} = H^T_{k+1}H_{k+1}P_{k+1|k}H^T_{k+1}H_{k+1}$$

Since $H^T_{k+1}H_{k+1}$ is positive definite, we further have

$$P^*_{k+1|k} = \alpha_k F_k P^*_{k|k} F^T_k + \Gamma_k Q_k \Gamma^T_k \ge P_{k+1|k} \tag{10}$$

Subtract (9) by (B.4) and use (10), we have

$$P^*_{k+1|k+1} - P_{k+1|k+1} = (I - K_{k+1}H_{k+1})\left(P^*_{k+1|k} - P_{k+1|k}\right)(I - K_{k+1}H_{k+1})^T \ge 0_{n\times n} \tag{12}$$

From (10)-(12), it is concluded that the fading factor satisfying (21) can guarantee the existence of a UBF. According to Theorem 4.1, this minimum fading factor can simultaneously result in the minimum upper bounds of covariance matrices of state prediction, filter residual and state estimate. Thus if (20) has only one solution to fading factor, then this solution will be the minimum fading factor. In the following part, the uniqueness of the solution to (20) will be explored.

First we explore the existence of the solution. Because $P^*_{k|k} > 0_{n\times n}$ and F_k is of full rank, we have $F_k P^*_{k|k} F^T_k > 0_{n\times n}$. Consider that H_{k+1} is of full row rank, we further obtain that

$$H_{k+1}F_k P^*_{k|k} F^T_k H^T_{k+1} > 0 \tag{E.1}$$

The two constraints of (20) can be easily satisfied. For example, in the case that α_k is $\max\{1, \frac{\lambda_{\max}\{V_{k+1} - H_{k+1}\Gamma_k Q_k \Gamma^T_k H^T_{k+1} - R_{k+1}\}}{\lambda_{\min}\{H_{k+1}F_k P^*_{k|k} F^T_k H^T_{k+1}\}}\}$, where $\lambda_{\max}\{\bullet\}$ and $\lambda_{\min}\{\bullet\}$ are the maximum and minimum eigenvalues of the corresponding matrix, the solution to (20) exists.

Then we need to verify that the solution to (20) is unique. Because the solution to (20) exists, the set Λ_k is not empty. If the set Λ_k only has one element, then that element will be the solution to (20).

Otherwise we can assume that $\alpha_k^{(1)} > 1$ and $\alpha_k^{(2)} > 1$ are two different solutions satisfying (20). Without loss of generality, we let $\alpha_k^{(1)} > \alpha_k^{(2)}$. There is

$$V_{k+1}^*\Big|_{\alpha_k=\alpha_1(k)} - V_{k+1}^*\Big|_{\alpha_k=\alpha_2(k)} = (\alpha_k^{(1)} - \alpha_k^{(2)}) H_{k+1} F_k P_{k|k}^* F_k^T H_{k+1}^T > 0_{m\times m} \tag{E.2}$$

(E.2) shows that $\alpha_k^{(2)}$ has the smaller upper bound than $\alpha_k^{(1)}$. It conflicts the assumption that both $\alpha_k^{(1)}$ and $\alpha_k^{(1)}$, as the solutions to (20), can result in the minimum upper bound. Therefore the optimization of (20) has one and only one solution. ■

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List of Figures and Tables

Table 1. Root-mean-square-error of the filtering results.

	k∈[1,50]		k∈[51,100]		k∈[101,150]		k∈[151,200]		k∈[201,250]	
	*x*1	*x*2	*x*1	*x*2	*x*1	*x*2	*x*1	*x*2	*x*1	*x*2
MUBF	6.96	5.05	12.04	15.14	13.69	17.81	17.30	20.73	18.96	22.07
ASF	6.57	5.60	13.41	34.52	20.07	38.01	64.63	80.01	28.55	24.90
FKF(alfa=1)	6.37	3.53	30.60	58.58	43.07	48.84	93.98	126.86	44.21	46.26
FKF(alfa=1.5)	6.57	5.60	13.41	34.52	20.07	38.01	64.63	80.01	28.55	24.90
FKF(alfa=3)	11.69	12.41	11.25	14.15	12.31	18.98	34.45	32.34	17.01	17.55
FKF(alfa=infinite)	20.32	19.04	18.34	19.41	20.43	18.59	17.44	22.27	22.45	24.24

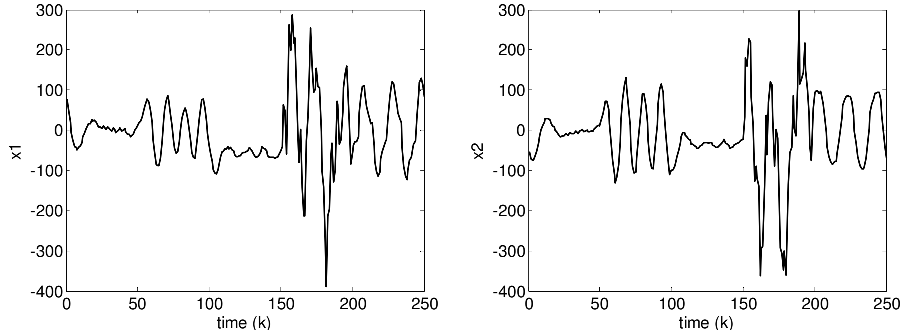

Figure 1. The true state to be estimated.

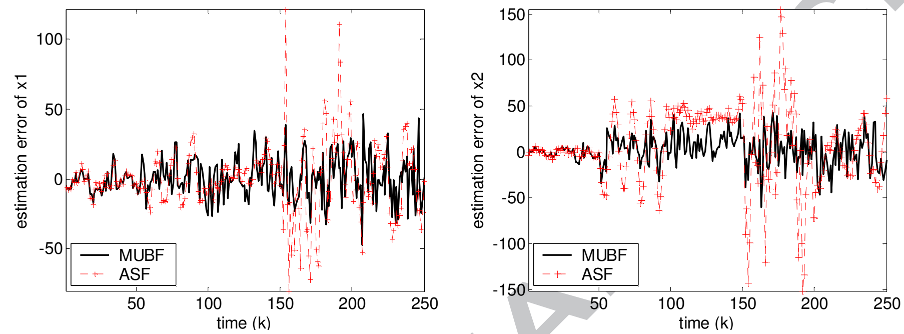

Figure 2. The state estimation errors by MUBF and ASF.

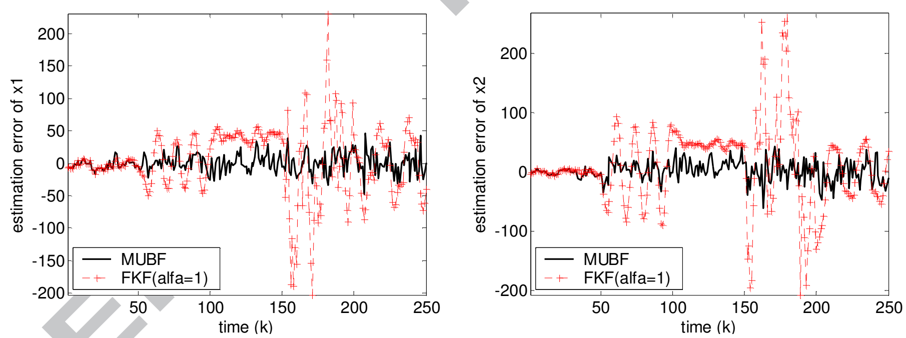

Figure 3. The state estimation errors by MUBF and FKF with alfa=1.

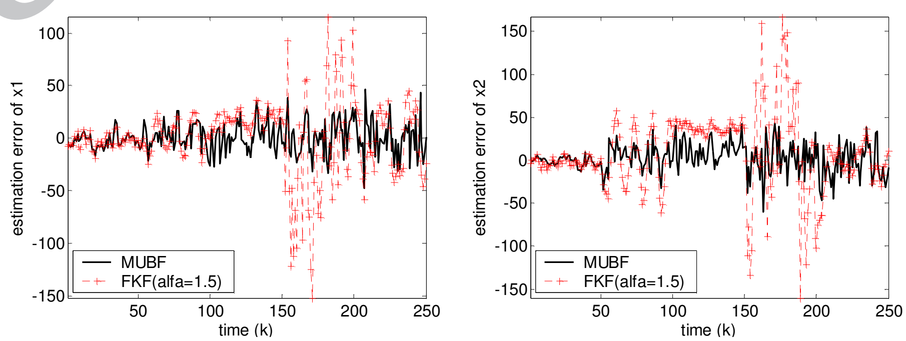

Figure 4. The state estimation errors by MUBF and FKF with alfa=1.5.

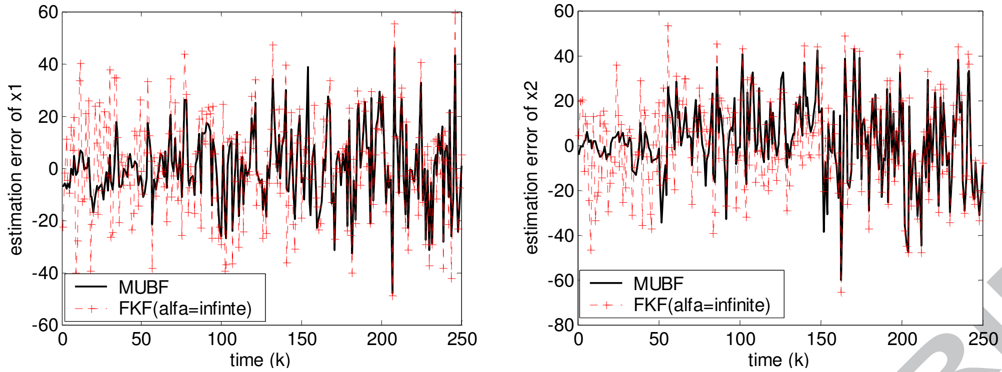

Figure 5. The state estimation errors by MUBF and FKF with alfa=∝.

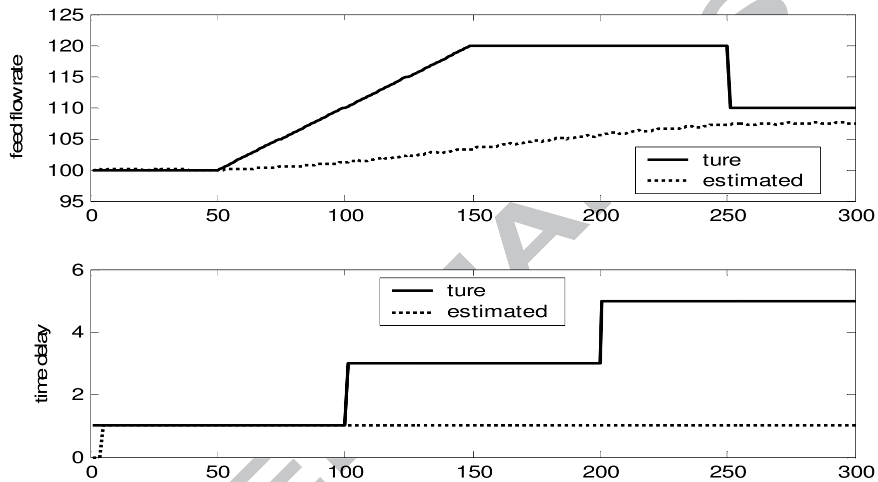

Figure 6. The filtering result of FKF (α=1).

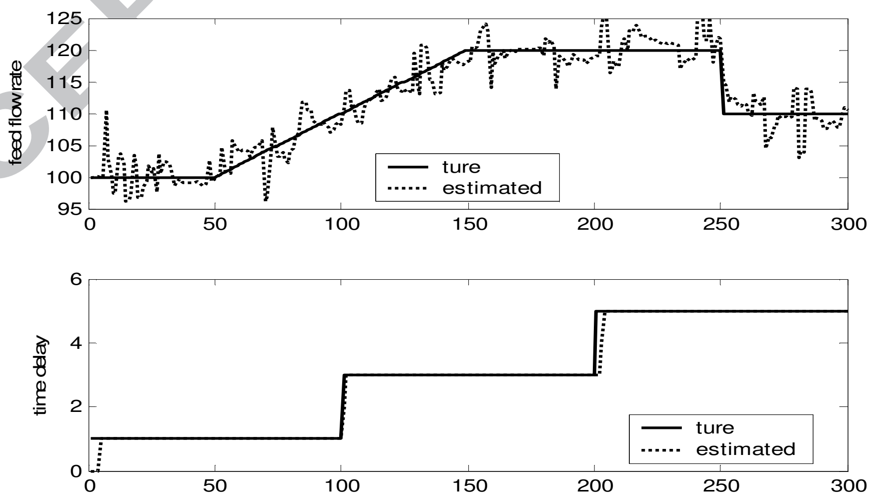

Figure 7. The filtering result of MUBF.

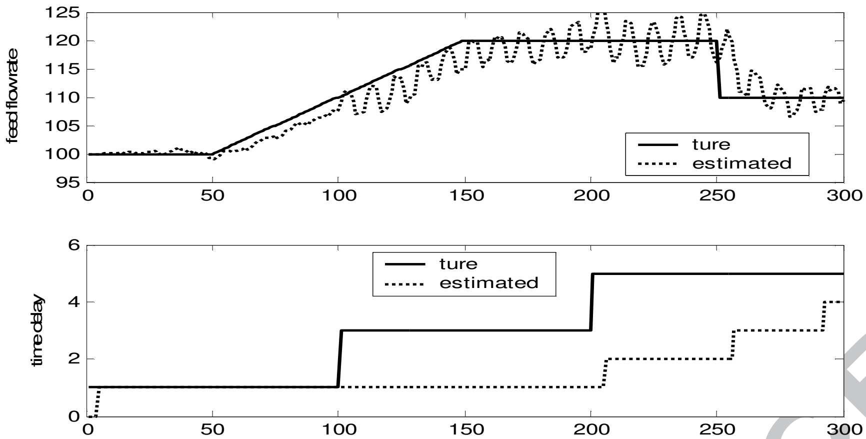

Figure 8. The filtering result of FKF (α=1.1).

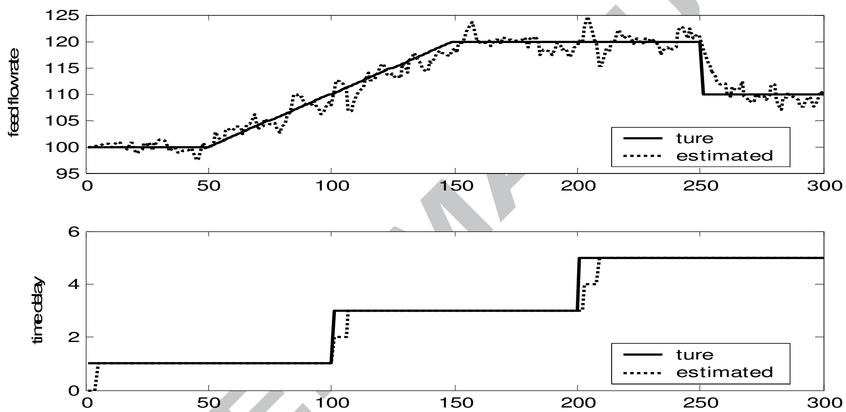

Figure 9. The filtering result of FKF (α=2).

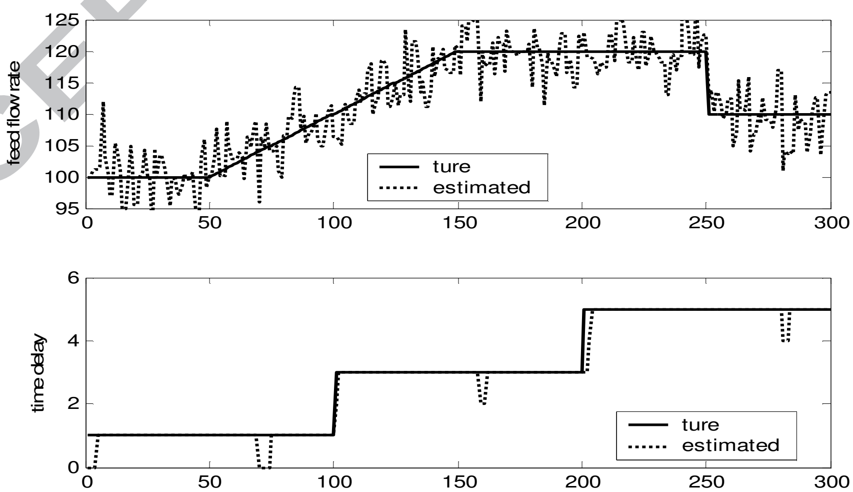

Figure 10. The filtering result of FKF (α=5).