Energy-Efficient Cooperative Transmission for Simultaneous Wireless Information and Power Transfer in Clustered Wireless Sensor Networks

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Abstract—This paper considers applying simultaneous wireless information and power transfer (SWIPT) technique to cooperative clustered wireless sensor networks, where energy-constrained relay nodes harvest the ambient radio-frequency (RF) signal and use the harvested energy to forward the packets from sources to destinations. To this end, we first formulate the energy-efficient cooperative transmission (eCotrans) problem for SWIPT in clustered wireless sensor networks as a non-convex constrained optimization problem. Then by exploiting fractional programming and dual decomposition, we develop a distributed iteration algorithm for power allocation, power splitting and relay selection to solve the non-convex optimization problem. We find that power splitting ratio plays an imperative role in relay selection. Our simulation results illustrate that the proposed algorithm can converge within a few iterations and the numerical analysis provides practical insights into the effect of various system parameters, such as the number of relay nodes, the inter-cluster distance and the maximum transmission power allowance, on energy efficiency and average harvested power.

Index Terms—Power allocation, cooperative relaying, wireless information and power transfer, clustered wireless sensor networks.

I. INTRODUCTION

Maximizing energy efficiency for data transmission becomes one of the most important design considerations in energy-constrained wireless sensor networks (WSNs). Moreover, in a clustered WSN, the relay nodes near cluster heads (CHs) will deplete their energy rapidly due to carrying out heavy tasks of data forwarding. Such nonuniform energy consumption may easily cause the network disconnected. It has been shown in [1] that cooperative transmission is more effective to balance energy consumption among nodes and improve energy efficiency of data transmission in WSNs. Recently, there have been some research efforts on developing cooperative schemes in clustered WSNs [2]–[5], in which sensors within a cluster relay data packets to nearby clusters using cooperative communication. A key element of cooperative transmission schemes is the selection and coordination of cooperative nodes.

In the meanwhile, energy harvesting technology has also been recognized as a promising cost-effective technique to maximize energy efficiency in WSNs. Unlike the conventional energy harvesting technique, which scavenges energy from the natural sources such as solar, wind and thermal, wireless power transfer (WPT) is an emerging energy harvesting technique, where sensors charge their batteries from electromagnetic radiation [6]. In WPT, green energy can be harvested through either strongly coupled magnetic resonances or radio frequency (RF) signals. The former requires that each sensor as an energy receiver to mount a coil tuned to resonate at exactly the same frequency as the coil on the energy transmitter [7]–[10]. However, in practice, sometimes it is difficult to mount a resonant coil in a small sensor. Moreover, energy transfer based on magnetic resonances is usually activated by near field induction from more powerful nodes (e.g., base stations and vehicles). Clearly, the application of this technology has some limitations in certain applications where there are no base stations near sensor nodes or the vehicle cannot travel or migrate very close to sensors, such as in wild forests and steep mountains.

On the other hand, compared to strongly coupled magnetic resonances, radio frequency (RF) signal can convey both energy and information simultaneously. Thus it is a promising energy source of wireless power transfer [11], since it can achieve both wireless information transmission and energy transfer, even in a hostile environment. Recently, a RF-based energy harvesting technique, called simultaneous wireless information and power transfer (SWIPT), becomes very appealing since it utilizes both information and energy carried by RF signals at the same time, and potentially offers great possibility to replenish the energy of sensor nodes. The core idea of SWIPT is that the receiver has two circuits to perform energy harvesting and information decoding separately [12].

SWIPT as an appealing energy harvesting technique has been applied to various types of wireless communication networks [13]–[21]. In [13], Lee, et. al. considered the application of SWIPT to cognitive radio networks. SWIPT for multi-antenna systems also attracts much attentions from researchers. In [14], Zhang, et. al. studied a three-node multiple-input multiple-output (MIMO) broadcasting system with SWIPT. Furthermore, Chen, et. al. extended the work in [14] by considering SWIPT in large-scale MIMO systems employing energy beamforming. In [16], Xu, et. al. studied a multiuser multiple-input single-output (MISO) broadcast SWIPT system. In [17], Chen, et. al.
analyzed the tradeoff of wireless energy and information transfer for limited-feedback multi-antenna systems. In the meanwhile, the application of SWIPT in orthogonal frequency division multiple access (OFDMA) systems has gained the attention in academia. The resource allocation algorithm was designed in [18] for energy efficient communication in OFDMA systems with SWIPT as an optimization problem. Subsequently, Zhou, et. al. [19] provided the optimal design for SWIPT in downlink multisuser orthogonal frequency division multiplexing (OFDM) systems.

Energy harvesting in wireless cooperative networks is particularly important as it can enable information relaying. In [20], the problem of SWIPT in an amplify-and-forward (AF) wireless cooperative network was studied. In [21], Ding, et. al. considered the application of SWIPT to wireless cooperative networks with one source-destination pair and multiple energy harvesting relays. However, these existing works did not consider how to optimally allocate transmit power and provide power splitting ratio to maximize energy efficiency. Compared to existing works in the literature, the contribution of this paper is that (i) we address the problem of energy efficient data transmission between clusters in WSNs by integrating RF-based SWIPT with cooperative relay, and (ii) we provide the optimal solution of power allocation, relay selection and power splitting to maximize system energy efficiency.

In this paper, therefore, we consider applying SWIPT to wirelessly charge the relay nodes with low energy in clustered WSNs. The superiority of this scheme lies in two aspects: (i) the network system can enjoy the benefit of cooperative transmission using intermediate sensors as relays in significantly saving energy; (ii) the relay nodes can be powered by the harvested energy as the energy compensation for data forwarding. This work aims at determining the optimal transmission power and relay selection, and finding the optimal power splitting ratio for energy harvesting and information decoding so that the system energy efficiency is maximized.

To this end, we first formulate the energy-efficient cooperative transmission (eCotrans) problem for SWIPT in clustered WSNs as a non-convex optimization problem constrained by the minimum harvested energy, the minimum system data rate, and the maximum transmission power. The non-convex optimization problem is solved by an iteration algorithm which combines nonlinear fractional programming and dual decomposition via appropriate objective function and optimization variable transformations. It is worth noting that although we employ a similar mathematical method to that in [18], [28] for formulating and solving the energy efficiency maximization problem, our work in this paper is significantly different from the work in [18], [28], which aimed at finding optimal policies of power allocation, subcarrier allocation and power splitting for energy efficiency optimization in OFDMA systems, instead of solving the problem of cooperative transmission with relay selection and energy harvesting in clustered WSNs.

Furthermore, we provide a distributed algorithm for power allocation, power splitting and relay selection. In particular, we find that power splitting ratio plays an imperative role in relay selection and it depends on the minimum harvested energy requirement. Finally, our simulation results demonstrate that the proposed algorithm can converge within a few iterations and its energy efficiency depends on the number of relay nodes and the inter-cluster distance. More importantly, we observe that the maximum allowed transmission power has a limited impact on average harvested energy. Compared to existing algorithms without adopting energy harvesting or energy efficiency maximizing, our proposed algorithm can achieve higher energy efficiency and more remaining energy.

II. SYSTEM AND COMMUNICATION MODELS

In this section, we first introduce the system model and communication model, and then formulate the optimization problem for energy-efficient resource allocation in a clustered WSN with SWIPT.

A. System Model

We consider a wireless sensor network consisting of multiple clusters of sensor nodes and a sink node as shown in Fig. 1, where sensor nodes are statically and randomly scattered over the sensing field. Each sensor node has a single antenna. The sink node is responsible for collecting data from all the sensor nodes. The nodes within the same cluster are distributed closely around the cluster head (CH), and can cooperate on signal transmission and/or reception. Suppose that the cluster head (CH) in a cluster (the source) wants to transmit data to the CH of its nearby cluster (the destination). Since the transmission distance is relatively long between clusters, the source can first broadcast the data to the member nodes in the cluster, select the “best” relay from a set of potential cluster member nodes, and then use this relay to aid the source-to-destination communication. Clearly, the transmission is the single-relay-selection cooperative communication scheme. It is worth noting that the destination here refers to the
In this paper, we adopt a practical dynamic power splitting (DPS) scheme, which is implemented by power splitting unit at the receiver, to enable the receiver to harvest energy and decode information from the same received signal at any time. The core idea of the DPS scheme is that a receiver $i$ dynamically splits the received signal into two power streams in the radio frequency (RF) front end with power splitting ratio $\rho_i^1$ and $\rho_i^2$ as shown in Fig. 2, which are used for decoding information and harvesting energy, respectively, where $0 \leq \rho_i^1 \leq 1$ and $0 \leq \rho_i^2 \leq 1$. In order to improve energy efficiency, we adopt the cooperative communication scheme in a decode-and-forward (DF) and time division relaying manner. As shown in Fig. 1, the energy harvesting cooperative transmission is carried out in two phase as follows.

**Phase 1:** The intracluster broadcasting transmission. When a CH has data to transmit, it first broadcasts a request-to-send (RTS) message to the cluster member nodes (CNs) within the same cluster to contend for the shared wireless channel. Once receiving the RTS message, the member nodes reply a clear-to-send (CTS) message to show being ready to communicate. These ready CNs also belonging to the receiving cluster form the set of candidate relays. After the RTS/CTS exchange, all candidate relay nodes will calculate their priority according to some predefined policies, which will be described in Section IV-D, based on the available channel state information fed back by RTS/CTS messages. The cluster member node with the higher priority will transmit and “win” the competition to serve as the relay for cooperative data transmission.

**Phase 2:** The intercluster cooperative transmission. After the relay is determined, the source sends out data to the relay/destination. The relay/destination first tries to direct the received data flow to the signal processing unit to decode and detect whether the minimum targeted data rate is satisfied, following the DPS approach. If the detection is successful and there is some energy left, the remaining signal flows will be directed to the energy harvesting unit, and the harvested energy will be used to support relay transmission. Then the source and relays will simultaneously transmit the packets to the destination (i.e., the CH of the receiving cluster).

The single-relay selection cooperative scheme is fully distributed and easy to implement. The underlying reasons are that it is much simpler than the multi-relay cooperation [2]. The former only selects one “best” relay to forward data while the latter requires the distributed space-time coding or beamforming. In particular, the selected relay can use the energy harvested to support relay transmission so as to avoid its energy being drained.

**B. Communication Model**

We consider two types of transmission modes for wireless communications: direct transmission mode (DT) and cooperative relay transmission mode (RT). Depending on whether the relay is helpful, each source may work in either the DT mode or the RT mode. Let $P_s$ and $P_r$ be the transmission power of source $s$ in Phase 1 and relay $r$ in Phase 2, respectively; $N_s$ and $N_r$ be the set of cluster member nodes (CNs) of source node $s$ (the CH in the transmitting cluster) and the destination node $d$ (the CH of the receiving cluster), respectively. Then the candidate relay set $\mathcal{N} = N_s \cap N_d$. We assume that all the links are symmetrical, i.e., the channel from node $i$ to node $j$ is the same as the channel from node $j$ to node $i$, and the channel from the source to the relay/destination follows quasi-static block fading. The channel is unchanged over the block time $T$ and independently and identically distributed from one block to the next, following a Rayleigh distribution. The use of such channels is motivated by prior research [12], [20]. We take a relatively short block duration compared to the minimum coherence time of the channel and interference such that both the channel and interference can be treated as unchanged during each block transmission. Let $h_{sd}$ and $h_{sr}$, $r \in \mathcal{N}$, denote the channel gains between source $s$ and destination $d$ and between source $s$ and its relay $r$, respectively, $h_{rd}$, $r \in \mathcal{N}$, denote the channel gain from source $s$ to destination $d$, and $\sigma_{sd}^2$, $\sigma_{sr}^2$ and $\sigma_{rd}^2$ denote the variances of the additive white Gaussian noise (AWGN) in the corresponding channels.
It is shown in [22], [23] that it is difficult to obtain perfect channel state information (CSI) due to noisy channel estimation and the unavoidable delay between the time channel estimation is performed and the time the estimation result is used for actual transmission. Therefore, we will consider imperfect CSI in this paper, i.e., the receiver knows the value of CSI, and both the transmitter and the receiver know the distribution of CSI, since transmitting CSI information would lead to extra overhead and considerable additional complexity.

To perform relay selection and power allocation, the source can obtain the channel gains by the feedback of the CTS message in Phase I via the dedicated control channel. To elaborate, in practical implementation, during the training period, i.e., RTS/CTS exchange in Phase I, before data transmission in the first time slot, the source transmits training signals by RTS message so that the relay and the destination measure the SR and SD channels and get the corresponding channel gains, respectively. The relay then transmits training signals in the second time slot to the destination so that the destination measures the RD channel and obtain its channel gain. The measured channel gains can be fed back to the source by CTS message on dedicated reverse control channels. Since all the links are assumed to be symmetrical, the source can obtain the SR and SD channel gains.

The normalized effective channel gains can be represented by $a_{sd} = |h_{sd}|^2/\sigma_{sd}^2$, $a_{sr} = |h_{sr}|^2/\sigma_{sr}^2$, and $a_{rd} = |h_{rd}|^2/\sigma_{rd}^2$, where $h_{ij} = k_{ij} L_{ij}^{-\alpha}$, $L_{ij}$ is the distance between transmitter $i$ and receiver $j$, $\alpha$ is a constant path loss exponent and $k_{ij}$ is a normalization constant depending on the radio propagation properties of the environment. As aforementioned, source $s$ via direct link would actively transmit data in both time slots while source $s$ via relay link would only transmit data in the first time slot. Thus, the end-to-end data rate from source $s$ to destination $d$ during the two phases is given by

$$R_{sd} = \begin{cases} B \log(1 + \rho^d a_{sd} P_s), & \text{DT mode} \\ \frac{B}{2} \min(\log(1 + \rho^d a_{sd} P_s + \rho^d a_{rd} P_r), \\ \log(1 + \rho^d a_{sr} P_s)), & \text{RT mode} \end{cases}$$

where $B$ denotes the base-band width and the rate is scaled by $\frac{1}{2}$ since the entire transmission takes two phases. A criterion to decide the working mode of the source in selective DF mode was given in [25], that is, using relay is advantageous when $\min(\rho^d a_{sr}, \rho^d a_{rd}) > \rho^d a_{sd}$. Otherwise, the relay keeps inactive in the relay phase.

In particular, the effect of best relay selection on the rate for the RT mode is reflected by the following two aspects.

(1) Best relay selection as a single relay cooperative scheme can avoid the complex mathematical expression for data rate since compared to multi-relay cooperative schemes, single-relay cooperation requires neither cooperative beamforming nor distributed space time coding [3].

(2) Best relay selection can ensure that the candidate relay that can provide the maximum data rate is always selected as the actual relay of source $s$ as shown in relay selection subalgorithm in Section IV.D. Moreover, the increase of the number of relays will increase the cooperation overhead and degrade the energy efficiency of cooperative communication, i.e., more cooperators may lead to less energy-efficiency [26]. This also motivates us to adopt the best relay selection.

We first consider the data rate $R_{sd}$ in the RT mode. Let $P_{sd}$ indicate the total transmission power between source $s$ and destination $d$ in the two phases. As mentioned in Section II-A, the harvested energy from the source is used by the relay as the energy compensation of data forwarding. This means that the harvested energy may not be enough for data forwarding to ensure the minimum data rate requirement, as shown in constraint C5. In this case, the relay has to consume part of its own energy for data forwarding. Therefore, the total consumed power should be the sum of the transmission powers of the source and the relay, i.e., $P_{s,d} = P_s + P_r$. If the harvested energy from the source is sufficient for data forwarding, then $P_{s,d} = P_s$. Clearly, the transmission power of relay $r$, $P_r$, includes two parts: one is the harvested power, denoted by $P_{r,harv}$, from the source, which is given by $P_{r,harv} = \eta \rho^r P_s |h_{sk}|^2$ [18], where $0 < \eta < 1$ is the energy conversion efficiency. The other is the power from the relay itself, denoted by $P_{r,own}$. Therefore, we have $P_r = P_{r,harv} + P_{r,own}$.

We can observe from (1) that the achievable rate is maximized when the amount of decoded information at the relay node is the same as the destination, i.e.,

$$1 + \rho^d a_{sd} P_s + \rho^d a_{rd} P_r = 1 + \rho^d a_{sr} P_s.$$  

Together with $P_{sd} = P_s + P_r$ and $P_r = P_{r,harv} + P_{r,own}$, we obtain

$$\begin{align*}
P_s &= \frac{\rho^d a_{sd}}{\rho^d a_{sr} + \rho^d a_{rd} - \rho^d a_{sd}} P_{sd}, \\
P_{r,own} &= \frac{\rho^d a_{sr} - \rho^d a_{sd} - \rho^d a_{rd} + \rho^d a_{sr} \rho^r P_s |h_{sk}|^2}{P_{sd}}.
\end{align*}$$

In the DT mode, we can easily obtain $P_s = P_{sd}$ and $P_r = 0$. Let $\lambda_{sd}$ be the equivalent channel gain given by

$$\begin{align*}
\lambda_{sd} &= \frac{\rho^d a_{sr} + \rho^d a_{rd} - \rho^d a_{sd}}{\rho^d a_{sd}}, & \text{RT mode}, \\
\lambda_{sd} &= \rho^d a_{sd}, & \text{DT mode}.
\end{align*}$$

Accordingly, by introducing a binary indicator $\vartheta_s$, which is 1 if source $s$ transmits data in the DT mode, and 0 in the RT mode, we can unify the data rate as

$$R_{sd} = \frac{B}{2} \left(1 + \vartheta_s\right) \log (1 + \lambda_{sd} P_s P_d).$$

We can observe from (5) that as the equivalent channel gain in (4) is directly proportional to power splitting ratio $\rho^d$, increasing $\rho^d$ can improve the unified data rate.

### III. Problem Formulation

In this section, we formulate the SWIPT based resource allocation optimization problem for cooperative transmission, aiming to maximize system energy efficiency.
A. Network Energy Efficiency

Let \( K \) denote the set of CHs in the network and \( |K| = K \), and \( \mathcal{N} \) denote the candidate relay set for source \( s \) and \( |\mathcal{N}| = N \). We assume \( t_{s,r} \) is a binary indicator which is 1 if relay node \( r \) is selected for forwarding data from source \( s \), and 0 otherwise. Next, we give the definition of the weighted system throughput.

**Definition 1 (Weighted System Throughput).** The weighted system throughput is defined as the weighted sum of the data rates that all the sources deliver to the destinations in the network and is given by

\[
U(P, \rho, T) = \sum_{s,d=1}^{K} \sum_{r=1}^{N} \alpha_r t_{s,r} R_{sd} \ [\text{bits/s}] 
\]  

(6)

where \( \mathcal{P} = \{P_{sd} \geq 0, \forall s, d \in K\} \) is the power allocation policy, \( \rho = \{\rho^1, \rho^2 \geq 0, \forall i \in K \cup \mathcal{N}\} \) is the power splitting policy and \( T = \{t_{s,r} \in \{0, 1\}, \forall s \in K, r \in \mathcal{N}\} \).

Let \( \alpha_r \) denote a non-negative weight which accounts for the priorities of different receivers to enforce certain fairness and is specified by the application layer. In practice, proportional fairness and max-min fairness can be achieved by varying the values of \( \alpha_r \) over time [24].

On the other hand, by considering the constant circuit power consumption and the inefficiency of power amplifier, we model the weighted power consumption as

\[
U_{TP}(P, \rho, T) = KP_{CH} + KNP_{CR} + \sum_{s,d=1}^{K} \sum_{r=1}^{N} \epsilon_t t_{s,r} P_{sd} 
\]

(7)

where \( P_{CH} > 0 \) and \( P_{CR} > 0 \) denote the constant circuit power consumption in the CH and relay node, respectively. Thus the first two terms indicate the total circuit power consumption in the \( K \) CHs and all relay nodes. The last term is the total power dissipation in the power amplifiers of all sources and the corresponding relays. \( \epsilon \geq 1 \) is a constant which accounts for the inefficiency of power amplifier in the source and relay nodes.

Next, we give the definition of weighted energy efficiency similar to [18].

**Definition 2 (Weighted Energy Efficiency).** The weighted energy efficiency of the considered system is defined as the total average number of bits successfully conveyed by the sources and relays to the destinations per Joule consumed energy and is given by

\[
U_{eff}(P, \rho, T) = \frac{U(P, \rho, T)}{U_{TP}(P, \rho, T)} 
\]

(8)

Compared to the energy efficiency in [18], the harvested energy at the receiver is not taken as the replenishment for the total system power consumption, which is because that from the whole network system point of view, the total energy of the whole system does not get replenished but is recycled and transferred from one node to another so as to achieve energy balance.

B. Optimization Formulation

As aforementioned in Section II, the power splitter splits the received signal \( y_r \) in \( \rho^1, \rho^2 \), such that the portion of the received signal, \( \sqrt{\rho^1} y_r \), is sent to the information decoding unit and the remaining signal strength, \( \sqrt{\rho^2} y_r \), drives the energy harvesting unit. Using the signal received at the input of the energy harvesting unit, similar to [20], the harvested energy at receiver \( k \) from transmitter \( s \) during a half of the block time, \( T/2 \), is given by

\[
Q_k = \eta \rho_k^E t_{s,k} P_s |h_{sk}|^2 (T/2) 
\]

(9)

where \( 0 < \eta < 1 \) is the energy conversion efficiency.

In this paper, we aim to provide the optimal power allocation policy \( \mathcal{P}^* \), power splitting policy \( \rho^* \), and relay selection policy \( T^* \) such that the weighted energy efficiency is maximized. To this end, the energy-efficient cooperative transmission (eCotrans) problem for SWIPT in clustered WSNs can be formulated as

\[
\text{OPT} - 1 \max U_{eff}(P, \rho, T) 
\]

(10)

Subject to

\[
C1 : Q_k + Q_{C,k} \geq E_{\min}^k, \forall k \in K, \\
C2 : \sum_{r=1}^{N} t_{s,r} P_{sd} \leq P_{\text{max}}^s, \forall s, d \in K \\
C3 : P_{CH} + \epsilon t_{s,r} P_{sd} \leq E_{\max} \\
C4 : \sum_{s=1}^{K} \sum_{r=1}^{N} t_{s,r} R_{sd} \geq R_{\min}^d, \forall d \in K, \\
C5 : \sum_{r=1}^{N} t_{s,r} R_{sd} \geq R_{m}^d, \forall s, d \in K' \\
C6 : t_{s,r} \in \{0, 1\}, \forall s \in K, r \in \mathcal{N}, \\
C7 : \sum_{r=1}^{N} t_{s,r} \leq 1, \forall s \in K, \quad C8 : \rho_{\min}^E \leq \rho_i^E \leq \rho_{\max}^E \\
C9 : \rho_{\min}^l \leq \rho_i^l \leq \rho_{\max}^l, \quad C10 : \rho_i^E + \rho_i^l \leq 1, \forall i
\]

where \( C1 \) is energy harvesting constraint which specifies that the sum of the harvested energy \( Q_k \) and the remaining energy \( Q_{C,k} \) should be bounded by the minimum required energy transferred to receiver \( k \), \( E_{\min}^k \). We assume \( E_{\min}^k \geq Q_{C,k} \) so as to guarantee the harvested energy \( Q_k \geq 0 \). Transmission power constraint \( C2 \) ensures that the power radiated by transmitter \( s \) is upper bounded by maximum transmission power \( P_{\text{max}}^s \). Power consumption constraint \( C3 \) restricts the maximum power supplied by the source for supporting the power consumption on its circuit and power amplifier to the maximum battery capacity \( E_{\max} \). \( C4 \) is a quality of service (QoS) constraint for the system that the aggregate network throughput should satisfy the minimum system data rate requirement, \( R_{\min} \).

Note that although \( R_{\min} \) is not an optimization variable in this paper, we can strike a balance between energy
efficiency and aggregate system throughput by varying its value. C5 is the minimum required data rate \( P_d \) for the delay constrained services of receiver \( d \), and is specified by the application layer, and \( K' \) denotes a set of receivers having delay constrained services. C6 and C7 are relay selection constraints which require that each relay node is only allocated to at most one source exclusively. C6 and C7 implicitly impose a fairness constraint, since each relay node is only allocated to at most one source exclusively. In other words, the relay allocated to a source is not allowed to forward the data from other sources. This implies that a weaker source also has a higher chance to be selected as a relay. C8 specifies that the power splitting ratio for harvesting energy is limited by the constant lower bound, \( \rho^{E}_{\text{min}} \), and upper bound, \( \rho^{E}_{\text{max}} \). These bounds reflect the limited capability of receivers in splitting the received power. \( \rho^{I}_{\text{min}} \) and \( \rho^{I}_{\text{max}} \) in C9 denote the constant lower and upper bounds of the power splitting ratio for decoding information, respectively, where \( \rho^{E}_{\text{min}} + \rho^{I}_{\text{min}} = 1 \) and \( \rho^{E}_{\text{max}} + \rho^{I}_{\text{max}} = 1 \). C10 reflects that the power splitting unit as shown in Fig. 2 is a passive device and no extra power gain can be achieved during the splitting process.

The key challenge in solving the optimization problem \( \text{OPT-1} \) in (10) is its lack of convexity due to the fractional form of the objective function and the couplings of optimization variables \( \{P, \rho, T\} \) in constraints C1-C5 and the objective function.

C. Transformation of Objection Function

We now transform the objective function in \( \text{OPT-1} \) problem in the fractional form into an equivalent one in the subtractive form via nonlinear fractional programming [27]. Without loss of generality, we define the maximum weighted energy efficiency \( q^* \) as

\[
q^* = \frac{U(P^*, \rho^*, T^*)}{U_{TP}(P^*, \rho^*, T^*)} = \max_{P, \rho, T} \frac{U(P, \rho, T)}{U_{TP}(P, \rho, T)} \tag{11}
\]

We introduce the following important theorem for solving the \( \text{OPT-1} \) problem in (10).

**Theorem 1.** The optimal resource allocation policies \( \{P^*, \rho^*, T^*\} \) achieves the maximum energy efficiency \( q^* \) if and only if

\[
\max_{P, \rho, T} [U(P, \rho, T) - q^* U_{TP}(P, \rho, T)] = U(P^*, \rho^*, T^*) - q^* U_{TP}(P^*, \rho^*, T^*) = 0 \tag{12}
\]

for \( U(P, \rho, T) > 0 \) and \( U_{TP}(P, \rho, T) > 0 \)

**Proof:** It follows from (6) and (7) that \( U(P, \rho, T) > 0 \) and \( U_{TP}(P, \rho, T) > 0 \) are satisfied and \( U_{\text{eff}}(P, \rho, T) \) is well defined. The remaining proof can be completed by following a similar approach to that in [28, Appendix A].

Theorem 1 reveals that for any optimization problem with an objective function in fractional form, there exists an equivalent objective function in subtractive form, e.g., \( U(P, \rho, T) - q^* U_{TP}(P, \rho, T) \) in the considered case, such that both problem formulations lead to the same optimal resource allocation policy.

D. Iterative Algorithm for Energy Efficiency Maximization

We now propose an iterative algorithm based on the Dinkelbach method [27] for solving the optimization problem \( \text{OPT-1} \) in (10) with the equivalent objective function \( U(P, \rho, T) - q U_{TP}(P, \rho, T) \). The proposed algorithm is described in Algorithm 1.

**Algorithm 1** Iterative algorithm for \( \text{OPT-1} \) problem

**Input:**
- \( \text{Iter}_{\text{max}} \): maximum number of iterations;
- \( \epsilon \): an infinitesimal number;
- \( q \): energy efficiency;
- \( j \): iterative index;

**Output:**
- \( \{P^*, \rho^*, T^*\} \): optimal resource allocation policy;
- \( q^* \): maximum energy efficiency;
- \( j \): iterative index;

1: \( q \leftarrow 0 \);
2: \( j \leftarrow 1 \);
3: while \( j \leq \text{Iter}_{\text{max}} \) do \{Main Loop\}
4: \( q \leftarrow U(P, \rho, T) - q U_{TP}(P, \rho, T) \).
5: if \( U(P, \rho, T) - q U_{TP}(P, \rho, T) < \epsilon \) then
6: \( q^* \leftarrow U(P, \rho, T) - \epsilon \).
7: end if
8: \( j \leftarrow j + 1 \);
9: end while

Algorithm 1 can be described briefly as follows. In each iteration of the main loop, we solve the transformed \( \text{OPT-2} \) problem in (13) for a given parameter \( q \) via dual decomposition and obtain an alternative optimal policy \( (P, \rho, T) \) of power allocation, power splitting and relay selection. Then we update parameter \( q \) and use it to solve the main loop problem in the next iteration until the condition \( U(P, \rho, T) - q U_{TP}(P, \rho, T) < \epsilon \) is satisfied, which implies that the iterative algorithm converges and the obtained allocation policy achieves optimum, i.e., \( (P, \rho, T) \rightarrow (P^*, \rho^*, T^*) \).

The transformed problem (\( \text{OPT-2} \)) for given energy efficiency \( q \) can be given by

\[
\text{OPT-2} \quad \max_{P, \rho, T} U(P, \rho, T) - q U_{TP}(P, \rho, T) \tag{13}
\]

Subject to constraints C1-C10.

Next, we verify the convergence of the iterative algorithm in Algorithm 1.

**Theorem 2.** The proposed algorithm of energy efficiency maximization in Algorithm 1 converges to the optimal energy efficiency if the optimization problem (13) can be solved in each iteration.

**Proof:** See Appendix A.
view. \( U(P, \rho, T) \) indicates the system profit due to data cooperative transmission while \( U_{RT}(P, \rho, T) \) represents the associated cost due to energy consumption. The optimal value of \( q \) indicates a scaling factor for balancing profit and cost.

Although the transformed optimization problem (OPT-2) has an equivalent objective function in subtractive form which is easier to handle, there are still two obstacles in tracking the problem. First, \( \rho_I^{t} \) and \( \rho_I^{s} \) are coupled with the power allocation variables in both the objective function and constraints C1, C4 and C5, which complicates the solution. Second, the binary constraint C6 on relay selection variables creates a disjoint feasible solution set and makes constraints C1-C5 become the combinatorial constraints, which is a hurdle for solving the OPT-2 problem.

In order to strike a balance between solution tractability and computational complexity, we handle the above issues in following two steps. In the first step, due to the integer constraint \( t_{s,r} \in \{0, 1\} \), problem OPT-1 is a mixed integer programming problem, which is in general non-convex and NP-hard. Thus, we first adopt the time-sharing relaxation technique that has been employed in [18], [29]–[32] to guarantee the convexity and tractability of the optimization problem. We relax the relay selection variable \( t_{s,r} \) in C6 to a real number between 0 and 1, i.e., \( 0 \leq t_{s,r} \leq 1 \). Then \( t_{s,r} \) can be interpreted as a time-sharing factor for the \( K \) sources to utilize relay node \( r \).

In the second step, we introduce a new auxiliary variable \( \tilde{P}_{sd} \), which is defined as \( \tilde{P}_{sd} = t_{s,r} P_{sd} \) and represents the actual transmitted power from source \( s \) to its destination \( d \) through relay node \( r \). In addition, we assume that the power splitting ratio for information decoding at relay \( r \) is the same as that at the corresponding destination \( d \), i.e., \( \rho_{I}^{t} = \rho_{I}^{d} \). This is justified since if \( \rho_{I}^{t} \neq \rho_{I}^{d} \), it can be observed from (4) that the update of \( \rho_{I}^{t} \) at relay node \( r \) depends on the update of \( \rho_{I}^{d} \) at its destination \( d \) in the proposed power splitting subalgorithm, and vice versa, which greatly increases the computation complexity and consumes much more energy for exchanging a large number of intermediate computation messages.

As for the suboptimality caused by the assumption, let \( \rho_{I}^{t} \) and \( \rho_{I}^{d} \) denote the suboptimal and optimal power splitting ratios for information decoding at destination \( d \), respectively, and we can obtain 
\[
\rho_{I}^{t} = \rho_{I}^{d} \left( a_{r,d} - a_{s,d} \right) / \rho_{I}^{d} \quad \text{for RT mode} \quad \text{and} \quad \rho_{I}^{t} = \rho_{I}^{d} \quad \text{for DT mode}.
\]

The latter is because the channel gain \( \lambda_{sd} \) is not related to \( \rho_I^d \). Clearly, for RT mode, when \( a_{r,d} \rightarrow a_{s,d} \), \( \rho_I^t \) will approximately equal \( \rho_I^{d} \). In practice, this case occurs frequently since the relay usually lies in the middle between the source and the destination, and the channel gain \( a_{r,d} \) between the relay and the destination is close to the channel gain \( a_{s,d} \) between the source and the destination.

Based on this assumption, we follow the approach in [18] and approximate the data rate as
\[
\tilde{R}_{sd} = \frac{B}{2} \left( 1 + \varrho_{s,d} \right) \log \left( \rho_I^t \lambda_{sd} \tilde{P}_{sd} / t_{s,r} \right)
\]
which is a tight approximation for high SINR, i.e., \( \lambda_{sd} \tilde{P}_{sd} \gg 1 \). Indeed, high SINR can be guaranteed since a minimum required system data rate \( R_{min} \) is set to guarantee a desired system data rate. \( \lambda_{sd} \) is defined as
\[
\lambda_{sd} = \begin{cases}
\frac{a_{s,d}}{a_{s,d} - a_{r,d}} & \text{RT mode,} \\
\frac{a_{r,d}}{a_{s,d} - a_{r,d}} & \text{DT mode.}
\end{cases}
\]
To remove the associated non-convexity, we can rewrite constraint C1 as
\[
C1' : \eta_{s,d} P_s |h_{s,d}|^2 (T/2) + \frac{Q_{C,d}}{\rho_I^d} \geq \frac{E_{min}}{\rho_I^d}.
\]

Next, we explore the convexity of the transformed OPT-2 problem with approximate data rate \( \tilde{R}_{sd} \) and auxiliary variable \( \tilde{P}_{sd} \).

**Theorem 3.** The transformed OPT-2 problem with constraints \( C1' - C10 \) is convex with respect to (w.r.t) the optimization variables \( \tilde{P}_{sd} \), \( \rho_I^d \) and \( t_{s,r} \).

**Proof:** See Appendix B.

Theorem 3 reveals that the transformed OPT-2 problem in (13) has a zero duality gap and satisfies the Slater’s constraint qualification. The zero-duality-gap result provides an avenue to obtain the optimal solution of the primal problem in (13) derived from its corresponding dual problem as will be seen later.

IV. DISTRIBUTED ALGORITHM FOR eCOTRANS PROBLEM

In this section, we solve the transformed OPT-2 problem with the approximated data rate \( \tilde{R}_{sd} \) in (14), relaxed constraint C4 and constraint \( C1' \).

**A. Dual Problem Formulation**

The resource allocation policy is derived via solving the dual problem of (13) with the approximated data rate function. For this purpose, we first give the Lagrangian function of the primal problem (13) by
\[
L(w, \eta, \mu, \nu, \phi, \varphi, P, \rho, T)
\]
where \( P_T = K P_{CH} + \sum_{k=1}^{K} \sum_{r=1}^{N_k} P_{CR} \), \( H_T = \eta |h_{sd}|^2 (T/2) \). Lagrangian multiplier \( w = [w_s, s = 1, \ldots, K]^T \) is for the inequalities of energy harvesting constraint C1, which denotes the prices for the individual minimum transferred power of harvested energy in C1. Lagrangian multiplier \( \mu = [\mu_{sd}, s, d = 1, \ldots, K]^T \) corresponds to transmission power constraint C2, which represents the price for the individual maximum transmission power. \( \nu = [\nu_{sd}, s, d = 1, \ldots, K]^T \) is Lagrangian multiplier for power consumption constraint C3, which indicates the price for the individual maximum power consumption. Lagrangian multiplier \( \eta = [\eta_k, b = 1, \ldots, K]^T \) is for the minimum required data rate requirement \( R_{\text{min}} \) of the system. Lagrangian multiplier \( \xi \) is for QoS constraint C4, representing the price for a relay corresponding to at most one source. Lagrangian multiplier \( \phi \) is Lagrangian multiplier for relay selection constraint C7, denoting the price for a relay to improve its data rate. On the other hand, the boundary constraints C8 and C9 on optimization variables are captured by the Karush-Kuhn-Tucker (KKT) conditions when deriving the resource allocation solution later.

The dual problem for the primal problem (13) is given by

\[
\min_{w, \eta, \mu, \nu, \xi, \phi, \rho, \mathcal{P}, \rho, \Upsilon} \max L(w, \eta, \mu, \nu, \phi, \xi, \mathcal{P}, \rho, \Upsilon) \tag{18}
\]

Based on zero-duality-gap result, we know that the solution of the OPT-2 problem in (13) can be derived from its dual problem in (18).

We use an iterative approach to solving the dual problem (18) as follows. In each iteration, given dual variables \( w, \eta, \mu, \nu, \phi, \xi \) and \( \phi \), we first calculate the primal variables \( \mathcal{P}, \rho \) and \( \Upsilon \) by applying the KKT conditions; Then by using the primal variables, we update the dual variables via the subgradient method. In the following, we give the corresponding distributed subalgorithms for power allocation, power splitting and relay selection.

B. Power Allocation Subalgorithm

Power allocation subalgorithm aims to determine the optimal transmission power at the source in Phase I and at the relay in Phase II, satisfying the constraints of maximum power consumption and minimum data rate requirement (QoS requirement). Using standard convex optimization techniques and the KKT conditions [33], for a given \( q \), in each iteration of the Dinkelbach method, the power allocation policy is given by

\[
P_{sd}^* = \left( \frac{(\alpha_s + \nu + \eta_d)B(1 + \phi_d)}{2 \ln 2(\Phi_{sd,r})} \right)^{\Phi_{sd,r}} \max_{x, b} \left[ \frac{(\alpha_s + \nu + \eta_d)B(1 + \phi_d)}{2 \ln 2(\Phi_{sd,r})} \right]^b \tag{19}
\]

where \( \Phi_{sd,r} = q e^{\mu_{sd}} + \nu_{sd} - w_s H_T \) and \( H_T = \eta |h_{sd}|^2 (T/2) \). Here operator \( \max \) is defined as \( \max(a, \min(x, b)) \). If source \( s \) and \( P_{sd}^* \) can be considered as a water vessel and its maximum water level, respectively, it is clear that different sources have different maximum water levels, and the power allocation in (19) has the form of multi-level water-filling, which can be interpreted as adaptively allocating transmission power according to a certain law and channel state. Usually, the link with good channel gain will always be allocated more power, that is, be filled more water up to its maximum water level in the vessel, in order to maximize transmission rate.

C. Power Splitting Subalgorithm

Power splitting subalgorithm aims at determining the optimal power splitting ratio at the receiver so as to guarantee that the harvested energy at the receiver is no less than the minimum required power transfer while the aggregated data rate is no less than the minimum system data rate requirement. In practice, the power split for energy harvesting and that for information decoding contradict with each other, that is, the increase of \( \rho_d^E \) will lead to the decrease of \( \rho_d^I \). The optimal power splitting policy can be obtained by solving the following maximization problem

\[
\max_{\rho_d^E} \left( \alpha_s + \nu + \eta_d \right) \left( t_{sd}^s - q \right) - \xi_d^R - \phi \left( \rho_d^I + \rho_d^E \right) \tag{20}
\]

Subject to

\[
C8: \rho_d^E \leq \rho_d^I \leq \rho_d^E
\]

\[
C9: \rho_d^I \leq \rho_d^E \leq \rho_d^I
\]

where \( w_d = E_{d}^{\min} - Q_{C,d} \).

By the KKT conditions [33], for a given \( q \), \( \rho_d^{I*} \) and \( \rho_d^{E*} \) are given by

\[
\rho_d^{I*} = \left[ \frac{B(1 + \phi_d)(\alpha_s + \nu + \eta_d)}{2 \ln 2 \phi_d} \right]^{\rho_d^{I*}} \tag{21}
\]

\[
\rho_d^{E*} = \left[ \frac{w_d(E_{d}^{\min} - Q_{C,d})}{\phi_d} \right]^{\rho_d^{E*}} \tag{22}
\]

We can observe from (21) that the power splitting ratio for information decoding, \( \rho_d^I \), is also a water-filling scheme and depends on the priority of the receiver via \( \alpha_s \), which implies that the receiver with high priority has to increase \( \rho_d^I \) to improve its data rate. Besides, Lagrange multiplier \( \nu \) forces the receiver to split larger ratio of power used to decode information in order to ensure that the aggregated network throughput satisfies the minimum system data rate requirement. On the other hand, \( E_{d}^{\min} \) and \( w_s \), require the receiver to increase the power splitting ratio for energy harvesting, \( \rho_d^E \), so as to meet the constraint of \( E_{d}^{\min} \).
D. Relay Selection Subalgorithm

The goal of the relay selection subalgorithm is to provide a relay selection criterion by which all overhearing nodes calculate their priority. The node with the highest priority will be selected as the relay node that cooperatively delivers data from the source. Thus, by using the standard convex technique [33] to solve the dual problem (18), relay node \( r \) is assigned to source \( s \) when the following selection criterion is satisfied

\[
t^*_r = \begin{cases} 
1, & \text{if } r = \arg \max_j M_{s,j} \\
0, & \text{otherwise} 
\end{cases} 
\] (23)

where

\[
M_{s,r} = \frac{B}{2} (\alpha_r + \nu + \eta_d) \log \left( \rho_r^t x_{sr} p_r^s \right) - \phi 
\] (24)

\( M_{s,r} \) can be regarded as the marginal benefit provided to the system when relay \( r \) is assigned to source \( s \). In other words, relay \( r \) is selected to cooperatively forward the data of source \( s \) if it can provide the maximum marginal benefit to the system, which implies that relay \( r \) has the highest priority to be selected among all candidate relay nodes of source \( s \). Besides, if relay \( r \) has a high priority, it will have a large value of \( \alpha_r \) and the resource allocator at the transmitter will have a higher preference to select relay \( r \). On the other hand, we can observe from (23) that although constraint relaxation is used in constraint C6 for facilitating the design of the resource allocation algorithm, the relay selection policy on each relay for the relaxed problem remains Boolean.

E. Lagrange Multiplier Update

In this subsection, we will solve the minimization problem at the high level in (18) by using the subgradient method which leads to the following Lagrange multiplier update

\[
w_s(t+1) = \left[ w_s(t) - \delta(t) \left( H_{sd} \tilde{P}_{sd} + \frac{Q_{C,d} - E_{d}^{\min}}{\rho_{d}^{\max}} \right) \right]^{+} \] (25)

\[
\eta_r(t+1) = \left[ \eta_r(t) + \delta(t) \left( t_{sr} R_{sd} - \tilde{R}_{d}^{\min} \right) \right]^{+} \] (26)

\[
\mu_{sd}(t+1) = \left[ \mu_{sd}(t) + \delta(t) \left( \tilde{P}_{sd} - \tilde{P}_{d}^{\max} \right) \right]^{+} \] (27)

\[
\nu_{sd}(t+1) = \left[ \nu_{sd}(t) + \delta(t) \left( P_{CH} + \varepsilon \tilde{P}_{sd} - E_{d}^{\max} \right) \right]^{+} \] (28)

\[
u(t+1) = \left[ \nu(t) - \delta(t) \left( \sum_{s=1}^{K} \sum_{r=1}^{N_s} t_{sr} R_{sd} - \tilde{R}_{d}^{\min} \right) \right]^{+} \] (29)

\[
\varphi(t+1) = \left[ \varphi(t) + \delta(t) \left( \rho_{d}^{E} + \rho_{d}^{I} - 1 \right) \right]^{+} \] (30)

where \( H_{sd} = H_T A_{sd,r} \), index \( t \geq 0 \) is the iteration index, and \( \delta(t) \) is a positive diminishing step size. Updating \( \varphi \) is not necessary as it has the same value for all nodes and does not affect the power splitting in (21) and (22) and the relay selection in (23). Therefore, we can simply set \( \varphi = 0 \) in each iteration. Indeed, in each iteration for solving the main loop problem, the master problem at the high level adjusts the Lagrange multipliers by (25)-(30). On the other hand, each subproblem at the lower level adjusts the water levels of (19), (21) and (22) and relay selection metric (23) by using the updated Lagrange multipliers. The procedure is repeated until convergence is achieved or the number of iterations reaches a predefined maximum number of iterations for the main loop, as shown in Fig. 3.

We now analyze the time complexity of the proposed iterative algorithm in Algorithm 1. It consists of two nested loops. The outer loop is to update the parameter \( q \) and can be proved to have a linear time complexity. On the other hand, the inner loop optimization problem is proved to be convex in Theorem 3, in other words, solving the inner loop optimization problem requires only a polynomial time complexity, i.e., the complexity is \( O(K \times N) \). As a result, the proposed algorithm has a polynomial time complexity, i.e., \( O(\text{Iter}_{\text{max}} \times K \times N) \).

V. SIMULATION AND DISCUSSIONS

In this section, we first verify the convergence of the proposed eCotrans algorithm. Furthermore, we compare and evaluate the performance of our solution for different parameters.

We assume that 50 cluster member nodes (CNs) are randomly located within a circular area within a radius of 120 meters. Here, we only use LEACH algorithm as an example to organize the clusters. Note that many other clustering protocols can also be used in our algorithm. We let \( \alpha_t = 2 \), \( \varepsilon = 5 \) and \( k_{ij} = 1 \). The channel gains are generated according to a Rayleigh fading model. Without loss of generality, we assume that all receivers have the same priority \( \alpha_r = 1 \), and all nodes have the same circuit power consumption, i.e., \( P_{C,H} = \tilde{P}_{C,H} = 10\text{dBm} \). We let \( \rho_{E}^{\min} = \rho_{I}^{\min} = 0 \) and \( \rho_{E}^{\max} = \rho_{I}^{\max} = 1 \). Moreover, to ensure fast convergence, the iteration step size adopted in
Lagrange multiplier updates is optimized via backtracking line search [33]. Other system parameters are listed in Table I.

A. Convergence and Performance Analysis

In this subsection, we study the convergence of the proposed eCotrans algorithm. Fig. 4 illustrates the evolution of energy efficiency for different maximum transmit power allowance $P_{\text{max}}$ and the number of relays $N$ in a cluster. By analyzing the results in Fig. 4, we can see that the energy efficiency increases with the number of iterations and then converges within 12 iterations in every considered scenario. Note that the number of iterations in Fig. 4 indicates only the main loop iterations for the Dinkelbach method, but not that for the gradient method. Another important observation is that the energy efficiency is directly proportional to $P_{\text{max}}$. This is justifiable since a higher transmit power allowance leads to the larger transmit power and data rate.

B. Impact of Relay Nodes on Energy Efficiency

In this subsection, we evaluate the impact of the number of candidate relay nodes in a cluster on energy efficiency under the DT and RT modes, respectively. To reflect the DT and RT modes, we let $a_{rd} = a_{rd}^d$ and $a_{rc} \geq a_{rd}$, which implies that if $a_{rd} > a_{rd}$, the CH works at the RT mode, otherwise, it works at the DT mode. We define $INR = a_{rd}/a_{rd}$ and let $INR$ be 0.5, 0.8, 1.2 and 1.5. Clearly, the first two ratio values indicate the DT mode is valid while the latter two values imply that the RT mode is available. We observe from Fig. 5 that compared to the DT mode, the energy efficiency of the proposed algorithm at the RT mode increases remarkably with the number of candidate relay nodes. However, the increase of energy efficiency becomes slower and finally stable with further increase of the number of relays. This is mainly due to that (i) it induces more collisions and energy consumption of all control messages such as RTS/CTS; (ii) the relay selection subalgorithm has to traverse more candidate relays and execute more iterations to find the optimal relay. Another important observation is that for the same transmission mode, the energy efficiency is directly proportional to the normalized effective channel gain.

C. Impact of Power Allowance on Average Harvested Energy

In this subsection, we explore the impact of the maximum allowed transmit power $P_{\text{max}}$ on the average harvested energy for different INR levels. Fig. 6 depicts the average harvested energy at the CH versus the maximum power allowance for different $INR$ levels. We set $INR$ to be 1.2, 1.5 and 2.0, which means that the CH works at the RT mode. It can be observed from Fig. 6 that in lower $P_{\text{max}}$, only a small portion of received energy is harvested by the CH for energy efficiency maximization. This is due to the fact that for small values of the transmit power allowance, the received power of the desired signal at the receivers may not be sufficiently large for simultaneous information decoding and energy harvesting. On the contrary, for the higher level of the transmit power allowance, the receiver has a higher tendency to split a larger proportion of the received power for energy harvesting until the amount of average harvested energy is saturated. This is because that once the constraints on the minimum required energy transfer to receiver $k$, $E_{k}^{\text{min}}$, and the minimum system data rate requirement, $R_{\text{min}}$, are satisfied, the transmitter stops increasing the transmit power for energy efficiency maximization. On the other hand, we can observe that a higher amount of energy is harvested by the receiver when the $INR$ level increases. As a result, splitting more received power for energy harvesting can enhance the system energy efficiency.

D. Impact of Inter-Cluster Distance on Energy Efficiency

In this subsection, we discuss how the distance $L_{ij}$ between the CH $i$ and the CH $j$ affects the energy efficiency under the different maximum transmit power allowance. In this case, we set the number of CN nodes in a cluster as $N = 8$ and let $P_{\text{max}}$ be $30\text{dBm}$ and $40\text{dBm}$. Fig. 7 reflects the evolution of energy efficiency with inter-cluster distance. It can be seen in Fig. 7 that when the inter-cluster distance $L_{ij}$ increases, the energy efficiency decreases, correspondingly. This is justified since when $L_{ij}$ increases, the proposed algorithm needs more sensor nodes to participate in cooperation to reach long transmitting distance for a given $P_{\text{max}}$, which leads
to more energy consumption. Another observation is that the energy efficiency at the RT mode is always larger than that at the DT mode as $L_{ij}$ increases. This is because that the long transmitting distance will hinder the direct transmission between the two CHs and even make the direct transmission invalid.

E. Comparison on Energy Efficiency and Remaining Energy

In this subsection, we compare the performance of the proposed eCotrans algorithm and several existing cooperative schemes, such as the eCocom scheme in [3], the coCoalition scheme in [4], the eCooperation algorithm in [5] and the coNetspa scheme in [21] in terms of energy efficiency and remaining energy for different maximum allowed transmit powers. The eCocom is an energy efficient selective single-relay cooperative scheme with physical-layer power control. The coCoalition is a cooperative communication scheme based on coalition formation game in clustered WSNs. The eCooperation is an energy-efficient cooperative transmission strategy using cooperative multi-input-multi-output (CMIMO) technique. The coNetspa scheme is an energy harvesting cooperative scheme for wireless information and power transfer in cooperative networks with spatially random relays. The first three schemes are energy efficient cooperative schemes, but without energy harvesting, while the coNetspa scheme involves the application of SWIPT to wireless cooperative networks, but it does not consider energy efficiency maximization.

In the simulation, we let $N = 8$ and $INR = 1.5$. We consider the scenario that all cluster member nodes (CNs) have the same amount of data to be sent and the same initial remaining energy. Fig. 8 illustrates the comparison of energy efficiency between the proposed eCotrans algorithm and the existing schemes. It can be observed that all the cooperative schemes have an increasing energy efficiency with the maximum transmit power allowance until the achieved energy efficiency gain attains its maximum in the high transmit power allowance region. However, our proposed eCotrans algorithm achieves the highest energy efficiency. This is justified since we employ the energy harvesting cooperative transmission, combining the optimal cooperative relay selection with the optimal power control and power splitting at the physical layer.

Fig. 9 depicts the remaining energy in the battery for receivers CH1, CN1, CN2 and CH3 as shown in Fig. 1 after transmitting the same amount of data by different cooperative schemes. It is observed that the proposed eCotrans algorithm has more remaining energy in the batteries of all receivers while also having the best remaining energy balance among receiver nodes. In particular, for the receivers CN1 and CN2, the superiority of the eCotrans algorithm is more obvious. The reason is that by using eCotrans algorithm, the receivers are able to harvest the energy from the received signals and replenish the harvested energy in the battery while forwarding the data from their upstream nodes. More importantly, although the CH3 node has more data to forward from its neighbor nodes as shown in Fig. 1, it still has more remaining energy in its battery. This is because the CH3 node is capable of harvesting the energy of ambient RF signals from the transmitters.

VI. CONCLUSIONS

In this paper, we consider applying SWIPT to cooperative clustered WSNs, where energy-constrained relay nodes avail the ambient RF signal and simultaneously harvest energy and process information to prolong their lifetime. Our goal is to provide the optimal policies for
power allocation and relay selection and determine the optimal power splitting ratio so that the system energy efficiency is maximized. To achieve this goal, we formulate the eCotrans problem as a non-convex constrained optimization problem. Furthermore, we propose a distributed iteration algorithm with closed-form transmission power, power splitting ratio and relay selection by exploiting dual decomposition. In particular, we find that power splitting ratio plays an imperative role in relay selection, however, it depends on the minimum harvested energy requirement. Our simulation results demonstrate that the proposed iterative algorithm converges within a small number of iterations. Compared to existing algorithms without energy harvesting or energy efficiency maximizing, our proposed iterative algorithm can achieve higher energy efficiency and more remaining energy.

**APPENDIX A**

**PROOF OF THEOREM 2**

We employ a similar approach to that in [27] [28] to prove the convergence of Algorithm 1. We first introduce two propositions to demonstrate the properties of the equivalent objective function in (13). For the sake of notational simplicity, we define $F$ as the set of feasible points of the optimization problem in (10) and let $F(q) = \max_{P,P,T} U(P,\rho,T) - q U^T P(P,\rho,T).$

**Proposition 1.** [28] $F(q)$ is a strictly monotonically decreasing function with respect to (w.r.t) $q$, i.e., $F(q') > F(q)$ if $q > q'$. 

**Proposition 2.** [28] Let $\{P',\rho',T'\} \in F$ be an arbitrary feasible solution and $q' = U_{U^T P}(P',\rho',T')$. Then $F(q') > 0$. 

Next, we prove the convergence of Algorithm 1. This proof includes two parts: the first part is to prove that energy efficiency $q$ increases with the number of iterations; the second part is to prove that if the number of iterations is large enough, then energy efficiency $q$ converges to the optimal $q^*$ such that it satisfies the optimality condition in Theorem 1, i.e., $F(q^*) = 0$. 

Let $\{P_n,\rho_n, T_n\}$ be the optimal resource allocation policies in the $n$-th iteration. We assume $q_n \neq q^*$ and $q_{n+1} \neq q^*$ represent the energy efficiency of the system in iterations $n$ and $n + 1$, respectively. By Proposition 2, we have $F(q_n) > 0$ and $F(q_{n+1}) > 0$. On the other hand, in the proposed Algorithm 1, we calculate $q_{n+1} = U(P_n,\rho_n, T_n)$. Thus we can compute $F(q_n)$ by 

$$F(q_n) = U(P_n,\rho_n, T_n) - q_n U^T P(P_n,\rho_n, T_n) = q_{n+1} U^T P(P_n,\rho_n, T_n) - q_n U^T P(P_n,\rho_n, T_n) = U^T P(P_n,\rho_n, T_n) (q_{n+1} - q_n)$$

Since $F(q_n) > 0$ and $U^T P(P_n,\rho_n, T_n) > 0$, it is not difficult to obtain $q_{n+1} > q_n$. That completes the proof of the first part. 

By $q_{n+1} > q_n$ and Proposition 1, we can obtain that $F(q_n)$ will eventually approach zero and satisfy the optimality condition in Theorem 1. That completes the proof the second part. 

**APPENDIX B**

**PROOF THEOREM 3**

We first prove that the transformed objective function $U(P,\rho,T) - q U^T P(P,\rho,T)$ is jointly concave w.r.t. the optimization variables $P_{sd}$, $\rho_{sd}$ and $t_{s,r}$. Then we show the convexity of constraints $C_{11} - C_{10}$. 

The concavity of the transformed objective function can be proved by the following steps. First, we consider the concavity of function $U(P,\rho,T)$ based on a relay selection w.r.t. the optimization variables $P_{sd}$, $\rho_{sd}$ and $\rho_{sd}$. 

For notational simplicity, we define a vector $x_{sd} = [P_{sd},\rho_{sd}^l,\rho_{sd}^E]$ and a function $f_{sd}(x_{sd}) = B(1 + \vartheta_s) \alpha_r \log_2 (\rho_{sd}^l P_{sd})$. Then we use $H(f_{sd}(x_{sd}))$ and $\tau_1$, $\tau_2$ and $\tau_3$ to denote the Hessian matrix of function $f_{sd}(x_{sd})$ and eigenvalues of $H(f_{sd}(x_{sd}))$, respectively. The Hessian matrix of function $f_{sd}(x_{sd})$ is given by 

$$H(f_{sd}(x_{sd})) = \begin{bmatrix} \tau_1 & 0 & 0 \\ 0 & \tau_2 & 0 \\ 0 & 0 & \tau_3 \end{bmatrix}$$

where $\tau_1 = -B(1 + \vartheta_s)\alpha_r 2\ln(2) 2 P_{sd}^2$ and $\tau_2 = -B(1 + \vartheta_s)\alpha_r 2\ln(2) 2 P_{sd}^2$ and $\tau_3 = 0$. Since $\tau_i \leq 0$, $i \in \{1,2,3\}$, $H(f_{sd}(x_{sd}))$ is a negative semi-definite matrix. In other words, function $f_{sd}(x_{sd})$ is jointly concave w.r.t. $P_{sd}$, $\rho_{sd}^l$ and $\rho_{sd}^E$. 

Then we can take the perspective transformation on $f_{sd}(x_{sd})$, which is given by 

$$u_{sd}(x_{sd}) = B(1 + \vartheta_s) \alpha_r \log_2 (\rho_{sd}^l P_{sd}) = \alpha_r t_{s,r} R_{sd}$$

It is shown in [33] that the perspective transformation preserves the concavity of the function. Thus function $u_{sd}(x_{sd})$ is jointly concave w.r.t. $P_{sd}$, $\rho_{sd}^l$, $\rho_{sd}^E$ and $t_{s,r}$. Function $U(P,\rho,T)$ is the sum of $u_{sd}(x_{sd})$ over indices $s, d$ and $r$, which preserves the concavity of the function [33]. 

In the following, we prove the convexity of function $U^T P(P,\rho,T)$ is an affine function of the variable $P_{sd}$, the function is convex w.r.t. the variable $P_{sd}$. Therefore, it is not difficult to obtain that the transformed objective function $U(P,\rho,T) - q U^T P(P,\rho,T)$ is jointly concave w.r.t. the optimization variables $P_{sd}$, $\rho_{sd}^l$, $\rho_{sd}^E$ and $t_{s,r}$. 

![Fig. 9. Remaining energy in the battery versus receivers for different cooperative schemes.](image)
Now, we verify the convexity of constraints $C_1' - C_{10}$. The left term of constraint $C_1'$ is linear, which implies that it is both convex and concave, and its right term is convex. Therefore, constraint $C_1'$ is convex. Since all the inequalities in constraints $C_2$ and $C_3$ are linear function of variable $P_{sd}$, clearly, the constraints $C_2 - C_3$ are convex. The relaxed constraint $C_6$ and constraints $C_{7} - C_{10}$ span a convex feasible set. As for constraints $C_4$ and $C_5$, it is easy to show that the constraints are convex due to the concavity of function $U(P, \rho, T)$.

As a result, the transformed OPT-2 problem is a convex optimization problem w.r.t. $P_{sd}$, $\rho_d^t$, $\rho_i^t$ and $t,s,r$.

REFERENCES


