Reliable Anchor-Based Sensor Localization in Irregular Areas

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Abstract—Localization is a fundamental problem in wireless sensor networks and its accuracy impacts the efficiency of location-aware protocols and applications, such as routing and storage. Most previous localization algorithms assume that sensors are distributed in regular areas without holes or obstacles, which often does not reflect real-world conditions, especially for outdoor deployment of wireless sensor networks. In this paper, we propose a novel scheme called Reliable Anchor-based Localization (RAL), which can greatly reduce the localization error due to the irregular deployment areas. We first provide theoretical analysis of the minimum hop length for uniformly distributed networks and then show its close approximation to empirical results, which can assist in the construction of a reliable minimal hop-length table offline. Using this table, we are able to tell whether a path is severely detoured and compute a more accurate average hop length as the basis for distance estimation. At runtime, the RAL scheme 1) utilizes the reliable minimal hop length from the table as the threshold to differentiate between reliable anchors and unreliable ones, and 2) allows each sensor to determine its position utilizing only distance constraints obtained from reliable anchors. The simulation results show that RAL can effectively filter out unreliable anchors and therefore improve the localization accuracy.

Index Terms—Wireless sensor networks, range-free localization, reliable anchor.

1 INTRODUCTION

Location awareness is becoming increasingly important in many sensor network applications and protocols, such as environment monitoring, vehicle tracking, and geographic-related routing protocols [1]. To acquire the location knowledge, it is cost-inhibitive to equip each sensor with a GPS receiver, and therefore only a limited number of special nodes in a network can be GPS-enabled, which are called anchors or beacons. Localization then becomes the problem to accurately infer locations of the “unknown” nodes with the assistance of only a few anchors. Localization techniques can be roughly classified as range based or range free. The range-based techniques depend on the measurements of internode distances or angles, by packet arrival time [2], signal strength [3], [4], the arrival angle of signals [5]. Their inadequacies are their requirement of the expensive per-sensor measuring equipments and their vulnerability to environmental interference. The range-free techniques, e.g., centroid approaches [6], multilateration [7], [8], and multidimensional scaling (MDS) [9], can remove this cost of ranging devices, by assuming the shortest paths between anchors and sensors proportionate to their euclidean distances.

Previous range-free localization work mainly assumes regular sensor deployment areas [10], i.e., sensors are uniformly and densely distributed in a convex region. However, this assumption does not hold when a sensor network is deployed in irregular areas with obstacles (or holes interchangeably), because the packet delivery path between two sensors can be distorted (or detoured) by obstacles and this shortest path distance is dramatically different from its geographical euclidean distance. These distorting obstacles are inevitable in natural areas such as valleys where sensors are deployed for habitat monitoring, as well as in urban areas where sensors can be separated by buildings. Therefore, when applying range-free techniques to the concave areas, the position estimates may in fact contain large errors [11]. One response to this irregular area problem is to partially ignore the erroneous distance information by using an improved multihop algorithm [12]. Yet, distorted anchor information can mislead accurate position estimates.

One way to improve the accuracy of localization would be to rule out distorted path information from some anchors, which however has two particular difficulties. First, because sensors do not have the global view of their network, they have no way of determining which path information is distorted and which is not. Second, while anchors can rely on the information that they receive from other anchors that are in an unobstructed (nondistorted) straight line path because they are able to determine their mutual reliability based on the calculation of an expected hop length, anchors and sensors cannot rely on each other in this way because sensors do not know their own locations and so cannot make an expected hop-length calculation.

In this paper, we propose a novel range-free scheme which we call Reliable Anchor-based Localization (RAL) and bears the following three advantages compared with other state-of-the-art work.
We introduce an accurate average hop-length estimation algorithm, which can both tolerate irregular radio propagation and the distortion effect of obstacles. Traditional offline hop-length estimation by Kleinrock equation [13] suffers from irregular radio propagation, while the online method of DV-Hop [7] is not robust to obstacle detours. Therefore, we first use the minimum hop length from a constructed reliable minimum hop length table to rule out unreliable anchors distorted by obstacles, and then apply the online hop-length estimation similar to DV-Hop using only reliable anchors.

- The adopted sensor localization algorithm residing on sensors can achieve high accuracy by heavily relying on information provided by reliable anchors. We first use the overlapping of ring and disk to get an approximate estimate about the sensor location. Here the ring-shaped constraints are from anchors within four hops, and the disk-shaped constraints are given by faraway anchors beyond four hops, whose paths are probably detoured by obstacles. With this approximate estimate about sensor location and some minimum hop length as a threshold, the sensor can tell which anchors are reliable and which are not. With recognized reliable anchors, the sensor can apply the basic MMSE multilateration [7] to derive an accurate location estimate.

- We reduce the requirement for high anchor density by the virtual anchor upgrade method. Potentially all range-free algorithms may suffer from low anchor density. Virtual anchors are selected for those sensors with good localization accuracy, which help alleviate the requirement for the high anchor density. We propose a computationally efficient way to select virtual anchors that can directly benefit sensor localization, especially those without enough number of reliable anchors.

We give theoretical analysis of the minimum hop length for sensor networks with uniform distribution and ideal radio transmission. However, empirical results show that the minimum hop length can be shortened due to radio irregularity. To improve localization accuracy, we should build the crucial minimum hop-length table offline that relates to the real sensor deployed environment, in a way to be adjusted empirically according to Degree Of Irregularity (DOI). Each sensor can consult with the table with network density (or interchangeably neighbor density) as its key. In this paper, we present a scheme that enables each sensor to independently estimate the number of neighboring sensors online and treat it as the network density, which makes the scheme more feasible in real applications.

We have conducted extensive experiments to test RAL in various network configurations. Simulation results show that RAL scheme can successfully filter out unreliable anchors. The obtained average hop length is much more close to the real case than the one applying all anchors’ information. Comparing RAL with other range-free schemes [7], [14], [12], we show that RAL can significantly improve localization accuracy in irregular networks and is insensitive to the global network density. Specifically, we show the robustness of RAL for sensors to obtain accurate location estimates when sensors are nonuniformly distributed, e.g., shape fitting, and DOIs are variant.

The rest of this paper is organized as follows: Section 2 describes work related to range-free localization techniques in detail. Section 3 provides a motivating scenario where the localization accuracy of sensors can be severely affected by irregular areas. In Section 4, we first illustrate the location problem due to obstacle detours, and then show the analysis on average and minimum hop length to assist identifying reliable anchors. Section 5 presents the RAL scheme. We present our simulation results in Section 6. Finally, Section 7 offers the conclusion.

2 Related Work

Range-free localization techniques, which depend solely on the content of received packets, use two main types of algorithms, centroid [6] and hop count to localize sensors. Centroid algorithms estimate the location of a sensor by calculating the centroid positions of proximate anchors. This requires a large number of anchors. In contrast, hop-count algorithms, such as APS [7] and Hop-TERRAIN [8], require only a small number of anchors. In APS, anchors broadcast to the entire network both their locations and a hop-count parameter initialized to one. The hop-count value is increased at every intermediate hop and each sensor records the minimum hop count as it receives it. In this way, all nodes in the network have information about the minimum number of hops to each anchor. Each anchor computes the euclidean distances (very accurate) to other anchors and estimates the average hop length which is then propagated out to nearby nodes. The average hop length that anchor $i$ computes is given by

$$h_i = \frac{1}{|A|} \sum_{j \in A} d_{ij},$$

where $d_{ij}$ denotes the euclidean distance between anchor $i$ and $j$, $h_{ij}$ denotes the hop count of the shortest path between them, and $A$ refers to the anchor set.

When the average hop-length information is available, sensor $k$ can estimate the shortest path length ($L_{ki}$) to anchor $i$ by

$$L_{ki} = h_{ik} \times h_{ln},$$

where $h_{ik}$ represents the hop count of the shortest path from anchor $i$ to sensor $k$, and $h_{ln}$ denotes the average hop length calculated using the anchor nearest to sensor $k$ (e.g., anchor $n$). To determine the position of sensor $k$, the multilateration technique based on all anchors can be used. Linear least squares or nonlinear least squares minimization solvers are commonly used to find the potential position $(x_k, y_k)$ of sensor $k$ that can minimize the following equation:

$$\{(x_k, y_k) = \arg \min_{x,y} \sum_{i \in A} (L_{ki} - d_{ki})^2,\n$$

where $d_{ki}$ denotes the euclidean distances between sensor $k$ and anchor $i$. 

$$\}$$

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This approach is accurate insofar only when the shortest paths between anchors and sensors approximate to their euclidean distances. However, there may be large errors in the distance estimates if the topology is not isotropic or contains a hole [11]. In order to alleviate the influence of holes, Shang et al. [14] suggest using only four nearest anchors assuming that the shortest paths to the nearest anchors may be less affected by irregularities, and this does produce good results in some cases but with a drawback of the possibility to falsely discard some good anchors which can improve the localization accuracy.

Lim and Hou [15] proposed a linear mapping method that transforms proximity measurements between sensor nodes into a geographic distance embedding space in anisotropic sensor networks. This transformation retains the topological information and reduces the effect of measurement noises on the estimation of geographic distances. APS [7] would estimate the position of sensor $S_1$ to be at $S_1'$ with regard to the 7-hop distance to $A_4$. The idea is that the resulting position of a sensor is derived by minimizing the deviation summation of the euclidean distances from estimated lengths in the shortest paths to all anchors. Thus, the coordinate that is calculated for sensor $S_1$ may be located in $S_1'$. Since the estimated length of the shortest path, in hops, to anchor $A_4$ is much larger than the actual value, to minimize the deviation, the estimated position of sensor $S_1$ is located far away from anchor $A_4$ (also far away from its real position).

In the hybrid approach proposed in [12], the position of sensor $S_1$ is in the area of intersection of all circles centered at anchors. The radii of circles are the estimated lengths of the shortest paths from sensor $S_1$ to anchors. A sampled point in the overlapping area that satisfies (4) is selected as the position of sensor $S_1$. In this way, the resulting position may be located in a position much closer to $S_1''$ since the position is still affected by the unreliable anchor $A_4$.

In both approaches, the accuracy of location estimation of a sensor node in an irregular network topology is compromised by the fact that they take into account information from all anchors even though some provided information is unreliable. In this scenario, the ideal localization for sensor $S_1$ can be derived by ignoring the unreliable anchor $A_4$. The potential position of sensor $S_1$ is still within the intersection area of all circles. However, only reliable anchors (i.e., anchor $A_1$, $A_2$, and $A_3$) are utilized in position estimation. Consequently, the calculated coordinates of sensor $S_1$ can be very close to its actual position. The issue in localization thus becomes how to distinguish reliable anchors from unreliable anchors to a sensor node. Note that an unreliable anchor to a sensor node may become reliable to other nodes located in different areas if no distorted paths are present.

3 Motivating Scenario

In this section, we present a scenario in which a network is deployed in an environment of obstacles and describe how two different typical non-RAL localization approaches attempt to localize sensors in irregular areas, i.e., multilateration-based APS [7] and a hybrid approach by Wang and Xiao [12]. Fig. 1 shows the network topology in which there are four anchors ($A_1$-$A_4$), seven sensors ($S_1$-$S_7$), and a large obstacle (in white). When anchors propagate their position information, each sensor records the physical location of each anchor and the hop count to it. It can be seen that the shortest paths from anchors $A_1$, $A_2$, and $A_3$ to sensor $S_1$ are approximately equal to their euclidean distances but the presence of the obstacle means that the shortest path from anchor $A_4$ to sensor $S_1$ requires a lengthy detour around the obstacle along the path ($A_4$ → $S_2$ → $S_3$ → $S_4$ → $S_5$ → $S_6$ → $S_7$ → $S_1$). A direct, unobstructed route between $A_4$ and $S_1$ might be, say, two hops. Clearly, the current 7-hop path does not give sensor $S_1$ an accurate estimate of the direct distance to anchor $A_4$.

4 Average and Minimum Hop-Length Estimation

In this section, we first identify the obstacle detour problem to solve in this paper, which shows that for a detoured path its hop-length estimates can be greatly diminished. This
In wireless sensor networks, assume that the wireless nodes are randomly distributed with the given density $\rho$. The area may contain obstacles or holes. The wireless sensor network can be modeled as an undirected graph $G = (V, E)$. The node set $V$ is a set of wireless nodes, including the anchor set $V_A$ and sensor set $V_S$. The edge set $E$ represents a set of virtual links between nodes. The geographic positions of anchors are pre-known. The target of our localization algorithm is to use anchors and edge set to infer the sensor positions. It is assumed that each sensor knows the network density. This constraint can be relaxed as discussed in Section 5.5.

Given a network as shown in Fig. 2a, the shortest path between anchors $A_1$ and $A_2$ is a 4-hop distance ($A_1 \rightarrow S_1 \rightarrow S_2 \rightarrow A_3 \rightarrow A_2$) in an isotropic network. Suppose that the transmission range of each node is $r$ and the euclidean distance between anchors $A_1$ and $A_2$ is $3r$. We can derive that the average hop length for this 4-hop path is $0.75r$. However, if there is an obstacle between anchor $A_1$ and $A_2$ as illustrated in Fig. 2b, the shortest path has to detour along the obstacle and becomes a 6-hop path ($A_1 \rightarrow S_3 \rightarrow A_4 \rightarrow S_4 \rightarrow S_5 \rightarrow S_6 \rightarrow A_2$). Now the calculated hop length is $0.5r$, which is much smaller than that in the isotropic network. Thus, in the absence of a global view of network topology, a sensor (or an anchor) can infer a smaller average hop length than a real one for the shortest path severely distorted by obstacles. This then raises the question of how to choose a hop-length threshold that would allow sensors to make accurately reliable determinations.

### 4.1 The Problem of Obstacle Detour

Assume that over a large area nodes are uniformly and randomly distributed with the given density $\rho$. The area may contain obstacles or holes. The wireless sensor network can be modeled as an undirected graph $G = (V, E)$. The node set $V$ is a set of wireless nodes, including the anchor set $V_A$ and sensor set $V_S$. The edge set $E$ represents a set of virtual links between nodes. The geographic positions of anchors are pre-known. The target of our localization algorithm is to use anchors and edge set to infer the sensor positions. It is assumed that each sensor knows the network density. This constraint can be relaxed as discussed in Section 5.5.

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### 4.2 Average Hop-Length Estimation

In wireless sensor networks, assume that the wireless communication range is $r$. Thus, the hop distance of a communication link has a maximum distance of $r$. Given the shortest path between node $k$ and anchor $i$ that contains $h_{ik} \cdot r$ hops, it is implied that node $k$ is at most $h_{ik} \cdot r$ distance from anchor $i$. However, given any two nodes, there may not be sufficient intermediate nodes for the shortest path to lie along the straight line between the source and destination [16]. Suppose the euclidean distance between sensor $k$ and anchor $i$ is $d_{ki}$, $d_{ki} \leq h_{ik} \cdot r$ and the average hop length is $h_l = d_{ki}/h_{ik}$. In [13], the following formula was derived to show the average hop length $h_l$ and how it is affected by the neighbor density $\rho$ for sensors uniformly distributed in an area.

$$h_l = r \left(1 + e^{-\rho} - \int_{-1}^{1} e^{-\frac{2(\arccos t - tv\sqrt{1 - t})}{t}} dt\right).$$

Given a shortest path with $h$ hops, all intermediate hops but the last one try to deliver the packet as far as possible. Suppose that the average hop length for the previous $h - 1$ hops is $h_{\text{normal}}$. According to [13], $h_{\text{normal}}$ should relate only to the neighbor density. The shortest path length $pl$ can be represented as follows:

$$pl = (h - 1) \times h_{\text{normal}} + h_{\text{lasthop}},$$

where $h_{\text{lasthop}}$ denotes the length of the last hop. The length of the last hop is usually unpredictable and within the range of $[0, r]$.

Then, the average hop length for the whole shortest path can be given by

$$hl = \frac{pl}{h} = h_{\text{normal}} + \frac{h_{\text{lasthop}} - h_{\text{normal}}}{h}.$$
4.3 Minimum Hop-Length Estimation

Although (7) provides much accurate calculation on the average hop length, it does not show a threshold regarding average hop length to differentiate detoured paths from normal ones. The following equations are for deriving the minimal hop length in isotropic networks. If the average hop length calculated from a path (assuming the path length is known from the position of its two ends) is lower than the minimal hop length in isotropic networks, we can tell that the path has been detoured by obstacles and thus its distance estimate is inaccurate.

The minimal hop length in networks with ideal circular radio range can be derived as follows:

\[ \begin{align*}
    p_{l_{\text{min}}} & : \text{Minimum path length in isotropic networks} \\
    h_{l_{\text{min}}} & : \text{Minimum hop length in isotropic networks} \\
    h_{l_{\text{min}}} &= \frac{p_{l_{\text{min}}}}{k} \\
    h &= 1, \quad h_{l_{\text{min}}} = 0; \\
    h &= 2, \quad h_{l_{\text{min}}} = \frac{r}{2}; \\
    h &= 3, \quad h_{l_{\text{min}}} = \frac{r + h}{3}, \text{where } h l \text{ is from (5)}; \\
    h &= k, \quad h_{l_{\text{min}}} = \frac{r + (k - 2)h}{k} \quad (k \geq 3).
\end{align*} \]

When the hop count \( h \) is one, the minimum hop length is zero, since the sensor can be very close to the anchor with \( p_{l_{\text{min}}} = 0 \). When \( h \) is two, the minimum hop length is \( \frac{r}{2} \), since the sensor can be at the fringe of the 1-hop transmission range \( r \) with \( p_{l_{\text{min}}} = r + h l \). When \( h \) is three, the minimum hop length is \( \frac{r + h l}{3} \), since the minimum distance for a 3-hop is at the outside edge of the first ring with \( p_{l_{\text{min}}} = r + h l \) (the ring width is the average hop length \( h l \)). When \( h \) is \( k \), the minimum hop length is \( \frac{r + (k - 2)h l}{k} \), since the minimum distance for a \( k \)-hop is at the outside edge of the \( (k - 2) \)th ring with \( p_{l_{\text{min}}} = r + (k - 2)h l \).

To test the effectiveness of the above theoretical minimum hop length in practical networks, we compare the theoretical result with experimental minimum hop length with various DOI values in Fig. 4. In experiments, we set the radio transmission range to be 1 while sensors are uniformly distributed in a 10 × 10 area without holes. In this setting, each sensor knows its position. All sensors broadcast messages in the network and we are easy to get the shortest physical distance among pairs of sensor nodes for a given hop count to compute the minimal hop length. Since the minimal hop length may be affected by the randomness of network topology, we calculated its average from 10 experiments. From Fig. 4, we can witness a good match between theoretical minimal hop length (from (8)) and its experimental result in isotropic networks with zero DOI value. However, the gap between them grows as the increase of DOI value. This is because with a large DOI value, there are non-negligible possibility for the presence of short range links with crossed distance much smaller than radius \( r \).

As a summary, three factors—sensor density (\( \rho \)), hop count (\( h \)), and radio irregularity (DOI)—all influence the minimal hop length in a wireless sensor network. Therefore, in real systems, we shall establish an offline table to reflect their combined influences on the minimum hop length, by statistically collecting sensor density and hop count information, which cannot be fully addressed by (8).

This offline table provides the minimum hop-length lookup capacity, indexed by two inputs (sensor density and hop count). For instance, we can build such table from experimental results in Fig. 5 where DOI = 0. The built empirical minimum hop-length table can help differentiate the undetoured path (with normal hop length) from detoured path (with abnormally diminished hop length), in concave networks with the presence of radio irregularity.

5 RELIABLE ANCHOR-BASED LOCALIZATION SCHEME

In this section, we describe our reliable anchor-based localization scheme, which we call RAL. Anchors have their pre-known accurate positions, such as equipped with GPS. We carry out the RAL scheme in five consecutive steps:

1. the propagation of anchor information;
2. the identification of reliable anchor pairs and the local calculation of average hop length;
3. the determination of the approximate area in which a sensor is located;
4. the identification of reliable anchors for a sensor; and
5. the distributed position estimate in a sensor by using reliable anchors.

These steps are performed at each node in a distributed fashion, with steps 1 and 2 conducted in anchors and steps 3-5 conducted in sensors.
Algorithms 1 and 2 present the pseudocodes for the RAL scheme. They depict the localization algorithms conducted in anchors and sensors, respectively. Please note that the parameter \( k \) represents the ID of a sensor or of an anchor. \( A \) is a node set containing all anchors. Each sensor or anchor contains a table that records the aforementioned reliable minimal hop length for a given shortest path hop count and neighbor density prior to the deployment of the wireless sensor network. At runtime, similar to other range-free localization approaches, RAL scheme ensures that anchors first broadcast their locations and a count is set to one. Each receiving node records the minimum hop count from an anchor and then floods outward the new hop count which is increased by one. Any packet containing a larger count value for an anchor is ignored. Thus, all nodes in the network obtain information as to the minimum number of hops to each anchor. We will introduce the average hop-length calculation procedure of anchors in Section 5.1. The detailed location estimation in sensors will be discussed in Sections 5.2 and 5.3. Section 5.4 will propose the virtual anchor concept to further reduce localization errors for sensors failed to find at least three reliable anchors. Section 5.5 will address the network density issue and how each sensor node can obtain neighbor density.

**Algorithm 1.** RAL algorithm resides in anchors

**Input:**
- \( k \): anchor ID, \( A \): the anchor set;
- \((x_i, y_i; h_{ki})\) where \(1 \leq i \leq |A| \) and \( i \neq k \): received position of anchor \( i \) and corresponding hop count to anchor \( k \);

**Output:**
- \( h_k \): the average hop length;

3: calculate the maximal possible length \( d_{\text{max}}^{k} \);
4: \( h_k = \frac{d_{\text{max}}^{k}}{h_{ki}} \);
5: threshold = lookup(density, \( h_{ki} \));
6: if \( h_{ki} > \text{threshold} \) then
7: \( \text{reliable}[i] = \text{true} \);
8: \( L_{ki} = h_k \times h_{ki} \);
9: end if
10: end for
11: return the point \((x_k, y_k)\) from \( PA_k \) satisfying (9);

**5.1 Calculation of Average Hop Length**

Equation (1) shows a simple way to derive the average hop length when an anchor receives the hop count and position information from other anchors. It assumes that information is obtained in a regular network topology yet as we can see in Fig. 2b where the shortest path between anchor \( A1 \) and \( A2 \) must detour around an obstacle in irregular topologies that are more close to real applications. Given anchor \( k \), its calculated average hop length \( h_k \) according to (1) in an irregular topology can in fact be much smaller than the real one. The smaller average hop length can lead to the estimated distance from a sensor to unreliable anchors longer than the actual distance while those to a reliable anchor will be shorter. To avoid the wrong distance estimation, in our calculation of the average hop length, we filter out information between anchor pairs who are incident on detoured paths. Fortunately, because anchor \( k \) knows the actual euclidean distances to all other anchors, it is possible to recognize a reliable anchor \( i \) by comparing \( h_{ki} = d_{ki}/h_{ki} \) with the threshold a constructed minimum hop-length table.

As shown in Algorithm 1, the function lookup(density, \( h_{ki} \)) searches the hop-length table for a reliable minimal hop length with the given neighbor density and hop count. If \( h_{ki} \) is larger than the threshold, anchor \( i \) is regarded as a reliable anchor to anchor \( k \). Otherwise, anchor \( i \) is eliminated from the average hop-length calculation. Algorithm 1 first allows that each anchor (i.e., anchor \( k \)) locally calculates the total hop counts and distances using all paths linked to itself. In our implementation, to increase accuracy we use all reliable anchor pairs to compute the average hop length. In this way, anchors can exchange the accumulated reliable hop counts and corresponding euclidean distances, derive the average hop length, and then distribute this information to nearby sensors by controlled flooding.

**5.2 Estimation of Approximate Location Area of Sensor**

When a sensor receives geographic information from anchors, it can use hop count and average hop length to independently estimate its likely location area. To accurately locate its position, the sensor should be able to identify reliable anchors and filter out unreliable ones who are offering confused path information. As a sensor has no knowledge of its location, to identify reliable anchors it must first obtain an approximate location estimate and then infer the maximal possible euclidean distances between itself and anchors.

To successfully obtain an approximate location area for a sensor, we propose a joint ring and overlapping circle
Algorithm 2 describes the operations carried out in a sensor to locally estimate its position. Only reliable anchors will be used for position estimate whose reliable value is true. A sensor $k$ computes the estimated path length $L_{ki}$ to the reliable anchor $i$ from the hop count $h_{ki}$ and correlated hop length $hl_i$. Sensor $k$ samples the point $(x_k, y_k)$ from the joint ring and overlapping circle area as in Fig. 6 that satisfies the following equation:

$$
(x_k, y_k) = \arg \min_{\text{reliable}[i]=true} \sum_{\text{reliable}[i]=true} |L_{ki} - d_{ki}|, \forall i \in A,
$$

where $d_{ki} = \sqrt{(x_k - x_i)^2 + (y_k - y_i)^2}$, $(x_k, y_k) \in PA_k$, and $PA_k$ is the likely location area of sensor $k$.

As we can see, Algorithm 2 can completely remove the impact of distorted paths from unreliable anchors in any irregular areas and yield accurate localization results. If a sensor cannot acquire sufficient number (less than 3) of reliable anchors, its location estimate could be far from its real location and thus cause a large estimation error. This may happen if anchors are sparsely or unevenly distributed in a sensor network area. However, we can create virtual anchors from precisely located sensors to accommodate this problem.

### 5.4 Virtual Anchor

When there are fewer than three reliable anchors identified by a sensor, the area in which it is likely to be located will be rather large and this could create a large estimation error. One solution to this problem is to find neighboring sensors that have been localized accurately and to use these as “virtual” anchors to refine the localization. To select sensors as eligible to act as virtual anchors, we first need to evaluate how accurately a sensor has located itself. In [12], this was evaluated by using the radius of the intersection area $PA_k$.

The radius $r_k$ of $PA_k$ is defined as the maximum distance between the estimated position $(x_k, y_k)$ to any other point $(x, y)$ within the area, that is,

$$
r_k = \max \sqrt{(x_k - x)^2 + (y_k - y)^2}, \forall (x, y) \in PA_k.
$$

When the radius $r_k$ is less than a given threshold, sensor $k$ is regarded as being eligible to act as a virtual anchor. However, such method requires the testing of each point in $PA_k$ and results in complicated computation.

In this paper, we propose to measure the location accuracy of a sensor node with regard to the possible distance variance from each reliable anchor. As shown in Fig. 7, suppose there are three reliable anchors available for sensor $S1$ and that the maximal and minimal possible distances of the sensor to reliable anchors have been calculated. The difference between the maximal ($d_{ki}^{\max}$) and minimal ($d_{ki}^{\min}$) possible distances to given anchors, say $d_{PA1}, d_{PA2}, d_{PA3}$, and $d_{PA4}$, reflects the size of the approximate area of location and consequently the localization accuracy.

In this paper, we utilize the following equation to calculate the average diameter of the intersection area:

$$
d_k = \frac{\sum_{\text{reliable}[i]} (d_{ki}^{\max} - d_{ki}^{\min})}{\text{number of reliable anchors}}.
$$
When the calculated average diameter $d_k$ of sensor $k$ is less than a given threshold, it is selected as a virtual anchor. Algorithm 3 depicts the virtual anchor selection process that resides on each sensor. A sensor that is eligible to act as a virtual anchor broadcasts its position information within a given range, for example, a 3-hop range to minimize the computation overhead. This process is similar to the anchor information propagation except that it is not propagated to the entire network. After a sensor receives the virtual anchor location and corresponding hop-count information, the virtual anchor will be treated as a normal anchor to further refine its location accuracy. The anchor set becomes $A \cup VA$ where $VA$ is a set constituting of all selected virtual anchors. The estimate position becomes

$$
(x_k, y_k) = \arg \min_{\text{reliable } i} \left[ L_{ki} - d_k \right], \quad \forall i \in A \cup VA. \quad (12)
$$

A sensor may become eligible to take on the role of a virtual anchor during the course of the position refinement iteration. Newly eligible virtual anchors can trigger the iteration process that helps other sensors to update their position estimates. To reduce computation overhead, for those sensors who estimate their positions from at least three reliable anchors or $d_k$ from (11) is small, they may simply bypass such iteration process.

**Algorithm 3.** Virtual anchor algorithm resides in sensors

**Input:**
- $k$: sensor ID, $m$: virtual anchor ID;
- vaThreshold: the upper threshold to be a virtual anchor;
- $VA$: the virtual anchor set;

**Output:**
- updated virtual anchor set: sensor $k$ can be added to $VA$;

1: sensor $k$ updates the intersection area $PA_k$ when receiving position information from virtual anchor $m$;
2: if virtual anchor $m$ is reliable then
3: sensor $k$ updates its location $(x_k, y_k)$ estimate according to (12) where $m$ is a member of $VA$;
4: end if
5: calculate $d_k$ of sensor $k$ according to (11);
6: if $d_k < \text{vaThreshold}$ then
7: sensor $k$ is eligible to be a virtual anchor;
8: $VA = \{k\}$;
9: broadcast the position info of sensor $k$ as a virtual anchor;
10: end if

### 5.5 Neighbor Density

As described in Section 4, we assume that each sensor have the knowledge of network density that will be utilized in the search of the reliable minimal hop length from the minimal hop-length table. However, in real-world sensor networks, the network density is usually unpredictable and such knowledge may not be pre-known. Taking this into account, we need to simplify our solution to practically let each sensor obtain the network density estimate, or in an approximate manner. A simple solution can use the number of neighboring nodes as the neighbor density for each sensor.

As was noted earlier in Fig. 3, the average hop length varies slightly when the neighbor densities are above 15. When the shortest path from an anchor to a sensor detours, the calculated average hop length is generally much smaller than the reliable minimal hop length no matter what the estimated neighbor density is, which means this simplified solution can still exclude unreliable anchors.

In our implementation, each sensor estimates the number of neighbors by counting the number of broadcast messages it has received from neighboring nodes or through a neighbor information exchange mechanism. When a sensor looks up the hop-length table, it utilizes the estimated neighbor density rather than the global network density. Later simulation results will confirm that RAL is not sensitive to the neighbor density parameter.

Reliable anchor-based localization with estimated neighbor density yields better results than other approaches.

### 6 Simulation Results

This section provides detailed quantitative comparisons of RAL with the localization algorithm described in [7, 14, 12], i.e., multilateration approach, four-nearest-anchors approach and hybrid approach. In the localization process, the multilateration approach will utilize information received from all anchors while four-nearest-anchors approach will only employ the four nearest anchors. In our simulation, nodes and anchors were uniformly and randomly distributed in three topologies: an isotropic square area, a C-shaped area and an S-shaped area. The transmission range of each sensor or anchor was normalized as 1. The square area was normalized with a side length 5. Thus, given the neighbor density that denotes the number of sensors within a unit circle, we can roughly estimate the total number of sensors distributed in a simulated area.

We evaluated different localization schemes in the metric of localization estimation error, that is, the distance between the estimated position and its true position of a sensor. The result of the multilateration method was obtained using linear least square followed by nonlinear least square optimization. We compare RAL with the hybrid approach [12] with and without iterations by using the virtual anchor technique. The plotted data illustrated in following figures represent the average result of 100 trials in randomly generated network topologies. In simulations, we set the network density of 36 and anchor number of 30 or otherwise stated.

#### 6.1 C-Shaped Topology

Fig. 8 shows the performance comparison in the C-shaped topology for four different approaches. The circles represent...
the real positions of nodes (solid circles for anchors and empty circles for sensors), and the lines represent the sensor estimation error. The average and maximal estimation errors of all schemes are listed in the figures. The comparison results show that the multilateration with all anchors (Fig. 8a) has the worst performance. This is because the position estimate will be affected by all anchors even though some of them in the C shape offer inaccurate information. Fig. 8b shows that using only four nearest anchors can give a better result since nearby anchors are less likely to be distorted by holes. The localization accuracy can be further improved by the hybrid approach as illustrated in Fig. 8c which selects points within a circling intersection area centered from all anchors. However, the distorted path may still affect the position estimation in the procedure for selecting points from the intersection area. The comparison shows that the RAL scheme (Fig. 8d) yields the best result since its localization is based on reliable anchors which can successfully eliminate the influence from distorted paths. Compared to the hybrid approach, RAL scheme can reduce the average localization error by \((\frac{0.46}{0.29} =) 36.96\) percent.

6.2 Reliable Anchor Selection

In the RAL scheme, both anchors and sensor nodes need to select only reliable anchors for accurate distance estimate in irregular areas. The reliable anchor selection can greatly affect the average hop-length calculation as denoted in Algorithm 1, and the estimate position accuracy of a sensor as denoted in Algorithm 2. Thus, we conducted simulations as the setting in Fig. 8 to validate the effectiveness of RAL of filtering out unreliable anchors. Fig. 9a illustrates the average hop length calculated by anchors with or without filtering out unreliable anchors. The flat line shows the global average result by filtering out unreliable anchors, which can reflect the real hop length. However, this empirical value is smaller than the theoretical average hop length from (5), partly because the last hop may decrease the average hop length to some degree (as shown in (7)). The fluctuating lines are the result calculated by each anchor locally. When an anchor calculated the average hop length based on information from all other anchors (totally 29), the value deviates tremendously from the flat line, which is normally much smaller than the real one. This is because detoured paths always introduce small hop length. Applying the RAL scheme, the locally computed average HopLength by each anchor is close to the flat line and can be used by neighboring sensors.

Fig. 9b illustrates the reliable anchor number obtained by each node. The first 30 nodes are anchors and the left nodes are sensors. Although a few nodes get 30 reliable anchors, most of them have much less reliable anchors in C-shaped sensor distribution areas.

6.3 Neighbor Density and DOI Impact

Now we investigate the impact of neighbor density on the localization estimation accuracy. The comparison results of three approaches were carried out in three topologies, which are shown in Fig. 10. The multilateration approach using only the four nearest anchors obtains almost the same estimation error for a given network topology no matter what the neighbor density is. This is because it always chooses four nearest anchors to locate a sensor. It can also be observed that in the other two localization schemes the localization error decreases slightly as the number of neighbors increases.
As shown in Fig. 10a, in an isotropic topology, multilateration using all anchors and RAL produces much better results than using only four nearest anchors. However, the performance of multilateration using four nearest anchors is much closer to reliable Anchor-based scheme because the nearest anchors can partly remove the distorted path information from faraway anchors in both C-shaped and S-shaped topology. In all three simulated topologies, reliable anchor-based scheme consistently outperforms the other two schemes, which we contribute to the identification of reliable anchors in the localization process.

We also show the effectiveness of our algorithm with the presence of different radio irregularity as illustrated in Fig. 11. It indicates that in C-shaped topology, the average estimation error of our approach slightly increases when the DOI is large. In S-shaped topology, we can observe simulation results where our RAL scheme can always outperform the other three.

### 6.4 Anchor Number Impact

In the simulation setting, we varied the number of anchors to observe its impact on localization errors. Fig. 12a shows that in an isotropic topology, multilateration using only four nearest anchors has the worst localization performance, the result of most anchors being eliminated in the localization process. In both isotropic and irregular environments, the RAL scheme benefits from increased anchors to have decreased overall estimation error because of more reliable anchors available.

Figs. 12b and 12c show the average localization errors in irregular areas, i.e., C-shaped and S-shaped topologies. In the
multilateration approach with all anchors, since the newly added anchors do not promise to provide a close approximation between the euclidean distance and shortest path distance, we see fluctuations in the average estimation error when increasing the number of anchors. In contrast, the precision of the other two localization schemes constantly improves as the number of anchors increases. When the number of anchors is up to 20 (around 10 percent of the total number of nodes), the localization error of RAL gradually reaches a stable value (between 0.3 and 0.2). RAL approach consistently outperforms the multilateration with only four nearest anchors with the increasing of anchor number.

6.5 S-Shaped Topology with Uniform and Nonuniform Distribution

Fig. 13 compares three localization approaches in the S-shaped topology and shows that the RAL approach is not sensitive to accuracy of the estimated neighbor density. In the simulation, each sensor counted its neighbors through the message exchange, regarding the number of neighbors as the local network density. To identify reliable anchors, a sensor needs to use the estimated density to look up the reliable minimal hop-length table. Without the knowledge of the global network density, the RAL scheme can practically get a rough estimate of the neighbor density, which in turn can effectively distinguish reliable anchors in the localization process. RAL can generate better results (smallest values for both the average and maximum localization errors) than the other two approaches.

Fig. 14 shows the simulation results for the hybrid approach, RAL with pre-known network density, RAL with estimated network density. The localization performance with estimated density using neighboring numbers is very close to the performance using pre-known network densities. These two curves are almost overlapping in different neighbor density environments. Both RAL schemes (with or without network density information) can achieve smaller average position estimation error than the hybrid approach did. The reason is that when the shortest path from an anchor to a sensor detours along an obstacle, the calculated hop length is generally much smaller than the reliable minimal hop length no matter what the neighbor density is estimated. Consequently, unreliable anchors can be ruled out for both RAL schemes in most cases. Thus, even without knowing the network density in advance, sensors can use the estimated neighbor density to accurately identify reliable anchors and achieve reliable location accuracy. This point is critical in practice where the network density is unavailable in randomly deployed networks.

Fig. 15 compares three localization approaches in the nonuniformly distributed S-shaped topology and shows that the RAL approach is not sensitive to nonuniform sensor distribution. From the empirical results, RAL can generate better results (smallest values for both the average and maximum localization errors) than the other two approaches. This satisfying performance of RAL in this network configuration with nonuniform sensor distribution is that when sensor density is above 20, the average hop length seldom changes, which can be seen from Fig. 3.

6.6 Virtual Anchor Impact

Virtual anchors can be applied to enhance the sensor location accuracy when a sensor cannot find at least three reliable anchors, which could happen given the sparse or uneven distribution of anchors. Fig. 16 shows the localization error when virtual anchors are selected to further reduce the localization errors. In this evaluation, sensors were uniformly deployed in a C-shaped topology. Only 15 initial anchors were randomly deployed in this area as shown in Fig. 16a. In the beginning, when the RAL scheme was performed with 15 anchors, a number of sensors have large localization errors either because they did not have three reliable anchors available or because the overlapping area was too large for accurate localization. Fig. 16b illustrates the position estimate deviation of each sensor without running the virtual anchor algorithm. The average estimation error was about 0.39. Fig. 16c shows the result when we carried out iterative location update process with newly selected virtual anchors. The threshold for defining a
virtual anchor was 0.2 in our simulation. The sensors that were eligible to act as virtual anchors broadcasted their position information within a predefined range (in this simulation, we set it to be a 3-hop range). Sensors were able to update their location estimates with improved accuracy by receiving broadcasted messages from virtual anchors. Newly eligible virtual anchors may appear throughout the iterative process. The average localization error decreases as the number of virtual anchors increases accumulatively. Compared with the hybrid method with iterations, our RAL method can finally allow more nodes to be treated as anchors (170) with minimized localization error. The reasons for this improvement could be: 1) in the proposed RAL, the unreliable anchor impact is removed in the average hop-length calculation; 2) RAL’s virtual anchor selection is more accurate. In our approach, we use the average diameter from different reliable anchors to indicate the accuracy of sensor’s localization. However, the hybrid approach uses the maximal length from the estimated location of a sensor in the potential overlapping area where we set the threshold to be 0.3. Fig. 16d shows the final location result in the iterative update process. Most sensors were eligible for virtual anchors and can locate themselves precisely. The average estimation error has been successfully reduced to 0.17.

We now investigate the effect of the initial anchor number on the RAL performance applying virtual anchor mechanism in the C-shaped topology. We varied the number of initial anchors from 3 to 60. As shown in Fig. 17, the RAL algorithm enhanced the location accuracy for sensors applying virtual anchor algorithm, which constantly performs better than the pure RAL algorithm. However, virtual anchors cannot replace initial anchors because a shortage of initial anchors means that none or only a few of sensors are eligible for the role of virtual anchors. The iteration process terminates quickly and not many sensors can benefit from it. Increased number of initial anchors is always better than using pure RAL algorithm since more reliable virtual anchors can be obtained with more real anchors.

7 CONCLUSION

In this paper, we propose a RAL scheme to reduce sensor localization errors in WSNs by eliminating the adverse impact of unreliable anchors detoured by obstacles. If the average hop length of the multihop path from an anchor is smaller than a predefined minimum hop length, we classify the anchor as a detoured and unreliable anchor. The RAL scheme can, therefore, minimize the impact of unreliable anchors in the online average hop-length estimation and sensor-anchor distance estimation. For the crucial minimum hop length, we have presented both theoretical analysis and

![Fig. 17. Localization errors with virtual anchor mechanism in the C-shaped topology.](image)
empirical results, showing the ability of our minimum hop-length table in reflecting the impact of radio irregularity, last hop distance, and different network density. The way to identify detoured path in the paper can be further extended to navigating mobile robots reliably in wireless sensor networks [17], e.g., avoiding the collision with obstacles.

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