Contactless 3D Fingerprint Reconstruction using Photometric Stereo

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Abstract: Fingerprint based personal identification is widely employed in civil and law-enforcement applications. Currently available contactless 3D fingerprint technologies either use multiple cameras or structured lighting to acquire 3D images of the fingerprints. Such approaches to recover 3D fingerprints increase the cost, complexity and bulk of the system and remain key limitations in popularity of 3D fingerprint systems, which otherwise can offer higher accuracy and hygiene than popular 2D fingerprint systems. The photometric stereo-based approach to 3D fingerprint recovery will require single camera and therefore significantly address the current limitations of 3D fingerprint sensing technologies. This has motivated us to investigate 3D fingerprint reconstruction using photometric stereo. In particular, this report describes the comparison of two models for recovering 3D fingerprints using photometric stereo: non-Lambertian and Lambertian approach. Our study reported in this document suggest that modelling of fingerprint skin by a linear reflection model (Lambertian), may be justified for its usage in 3D fingerprint reconstruction as it can provide superior or similar accuracy, in addition to being computationally simpler to the non-Lambertian model for 3D fingerprint recovery considered in this work.

1. Introduction

The acquisition, analysis and recognition of live fingerprints is widely considered to be an active area of research in the biometrics community. Most of the criminal and civilian fingerprint systems today accept live-scan images that can be directly acquired from the finger-scan sensors. Conventional fingerprint acquisition requires touching or rolling of fingers against a rigid sensing surface. Such touch-based sensing is tedious, time-consuming and often results in partial or degraded images due to improper finger placement, skin deformation, slippage, smearing or sensor surface noise. With the significant growth in the demand for stringent security, the touch-based fingerprint technology is facing a number of such challenges and therefore contactless 3D fingerprint systems have recently emerged.

The contactless live fingerprinting is essentially a 3D surface reconstruction problem and currently available solutions employ vision based reconstruction techniques such as shape from silhouette or stereo vision using structured lighting. Reconstruction of 3D fingerprints using multi-view shape from silhouette requires
multiple cameras, for example the commercial system from TBS (surround imager) [19] uses five cameras, while a specialized projector and high-speed camera is required for the structured light based 3D fingerprint systems such as those employed in [20]. Despite such complexity, both of these approaches have demonstrated to outperform the current 2D fingerprint technologies in achievable recognition accuracy and response time. The main obstacle to these emerging touchless 3D fingerprint technologies, in replacing the conventional fingerprint systems, is their cost and bulk, which is mainly contributed from the usage of structured lighting system or multiple cameras. Furthermore, these technologies have not been able to exploit other live surface parameters, e.g. surface normal vectors, scattering parameters, refraction parameters, etc., which can also contribute to the improved fingerprint reconstruction and recognition results. Our ongoing work therefore proposes to develop a new approach for the touchless fingerprint identification using photometric stereo which can provide low-cost, faster, and more accurate alternative to the conventional touch-based fingerprint identification.

Our work described in this technical report investigates specific problem of 3D fingerprint reconstruction using photometric stereo (PS). We particularly compare non-Lambertian and Lambertian models for the 3D fingerprint reconstruction. Such comparison will help to ascertain suitability of using fingerprint reconstruction models during the live fingerprint recognition.

2. 3D Fingerprint Reconstruction using Photometric Stereo

Traditional photometric approaches uses shading cues from three or more known lighting conditions and compute the depth of surface points by solving linear Lambertian equations. It is well known that traditional PS method is such a simple 3D reconstruction technique: by giving three or more known lighting conditions, surface normal vectors and albedo can be obtained by solving a group of linear equations. However, traditional PS method may be limited to Lambertian surface, i.e., the surface that has ideal diffuse reflection and abides by Lambert Reflection Law [5]-[7]. Recently it has been introduced in skin recovery where human skin is assumed to be Lambertian surface, for example, authors in [7]-[8] use traditional PS method to detect skin lesion. In fact, skin is a kind of translucent material whose reflectance model contains a lot of considerable multiple scattering and specular reflections, as illustrated in figure 1, therefore it is judicious to suspect that simple skin modelling by a linear reflection model like Lambert may not produce accurate results, especially in the case where high reconstructing
precision of fingerprints are demanded. In order to model the human skin more precisely, Georghiades [9] has introduced a non-Lambertian reflectance model, say Torrance and Sparrow (TS) model, into uncalibrated PS method to calculate reflectance parameters of human skin and to reduce the negative effects of generalized bas-relief (GBR) [10]. TS is a physically-based model. It assumes that reflectance consists of two components: a) Lambertian lobe at a particular position on the surface and b) purely surface scattering component. In comparison, to human skin and fingerprints which have translucent surfaces, the Hanrahan-Krueger model (HK) considers the multiple scattering under layer-structure of skin and that make it more reasonable [4] to explore its usage in 3D fingerprint identification.

2.1 3D Fingerprint Reconstruction using Photometric Stereo

One of the promising models for the subsurface scattering The fingerprint skin is a kind of translucent material whose reflectance model in layered surfaces is based on one dimension linear transport theory. The basic idea is that the amount of light reflected by a material that exhibits subsurface scattering is calculated by summing the amount of light reflected by each layer times the percentage of light that actually reaches that layer. In this model, the skin is modelled as a two-layer material corresponding to the epidermis and the dermis, each with different reflectance parameters that determine how light reflects from that layer.

2.2 Overview of Methodology

![Figure 1: Reflection and multiple scattering in two-layer skin model [4].](image)

In the original HK model [4], each layer is parameterized by the absorption cross section $\sigma_a$, the scattering cross section $\sigma_s$, the thickness of the layer $d$, and the mean cosine $g$ of the phase function. The parameter $p$ determines in which direction the light is likely to scatter as (1), where $\phi$ is the angle between the light direction and
the view direction:
\[
p(\phi, g) = \frac{1}{4\pi} \cdot \frac{1 - g^2}{4\pi(1 + g^2 - 2g\cos\phi)^{3/2}}
\] (1)

Additionally, the total cross section \( \sigma_t \), the optical depth \( \tau_d \) and the albedo \( \zeta \) can be expressed as: \( \sigma_t = \sigma_s + \sigma_a \), \( \tau_d = \sigma_t \cdot d \), \( \zeta = \sigma_s / \sigma_t \). The total cross section represents the expected number of either absorptions or scatterings per unit length. The optical depth is used to determine how much light is attenuated from the top to the bottom of the layer. And the albedo represents the percentage of interactions that result in scattering. Accordingly, higher albedo indicates that the material is more reflective, and a lower albedo indicates that the material absorbs most of the incident light. Besides, Hanrahan and Krueger also noted that the reflectance curve from multiple scattering events follows the same shape as the curve for a single scattering event, meaning that more scattering events tend to distribute light diffusely.

In this investigation, to simplify the HK model, we use a Lambertian term \( L_m \) to approximate the multiple scattering \( L_{r2}, L_{r3}, \ldots, L_{rm} \), which is added to a single scattering result.

\[
L_m = L_{r2} + L_{r3} + \cdots + L_{rm} = \rho \cos \theta_i
\] (2)

where \( \rho \) is an defined albedo, which determines the amount of diffuse light caused by multiple scattering, and \( \theta_i \) is incidence angle, i.e. the angle between the normal vector and the vector of light direction. Then the revised HK model can be written as:

\[
L_r(\theta_r, \varphi_r) = \zeta T_{12} T_{21} \frac{p(\phi, g) \cos \theta_i}{\cos \theta_i + \cos \theta_r} \left(1 - e^{-\tau_d \left(\frac{1}{\cos \theta_i} + \frac{1}{\cos \theta_r}\right)}\right) L_i(\theta_i, \varphi_i) + L_m
\] (3)

Given the incident lights from some direction \( L_i(\theta_i, \varphi_i) \), the amount of reflected light \( L_r(\theta_r, \varphi_r) \) can be calculated through Equation (3), where \( p(\phi, g) \) has the expression in (1), \( T_{12} \) and \( T_{21} \) are the Fresnel transmittance terms for entering and leaving the surface respectively, they are assumed to be constant over the whole surface as well as \( g \). Equation (3) is only calculated for epidermis. We ignore scattering in dermis, since this factor do minus contributions to the final image and too many unknown parameters also will reduce the estimation accuracy of main parameters. Notice that \( \cos \theta_i \) and \( \cos \theta_r \) in Equation (3) can be rewritten in the form of normal vector \( \mathbf{n} \)'s inner product of light direction vector\( ^\dagger \) \( \mathbf{l} = (l_x, l_y, l_z) \) and the view direction \( \mathbf{z} = (z_x, z_y, z_z) \):

* \((\theta_i, \varphi_i)\) are angles of incidence, likewise, \((\theta_r, \varphi_r)\) are angles of reflection.

\( ^\dagger \) Here, \( \mathbf{n} = (n_x, n_y, n_z) \), \( \mathbf{l} \) and \( \mathbf{z} \) are normalized vectors.
$$\cos \theta_i = l \cdot n, \cos \theta_r = z \cdot n$$  \hspace{1cm} (4)

Each of the surface point is expected to have its unique normal vector, which means the corresponding quadruple \((\theta_i, \phi_i; \theta_r, \phi_r)\) is unique. In order to clarify this representation two superscripts are used to mark those point-unique parameters with the first superscript indicating the order of the surface point and the second one indicating the number of the light source. For example, \(L_r^{j,k}\) represents \(L_r(\theta_r, \phi_r)\) of the \(jth\) surface point under the illumination of the \(kth\) light, \(n^i\) refers to normal vector of the \(jth\) surface point, while \(l^k\) represents the direction vector of the \(kth\) light. Then we can formulate the simultaneous recovery to the following nonlinear optimization problem:

$$\arg \min_x E(x),$$

where \(E(x) = \sum_{j,k} (L_r^{j,k} - I^{j,k})^2\) \hspace{1cm} (5)

where \(I^{j,k}\) represents the pixel intensity on the \(kth\) image of the \(jth\) surface point, and \(x\) is a 10-dimensional containing all the unknown parameters to be estimated:

$$x = (n^i_x, n^i_y, n^i_z, \rho^j, d^j, \sigma_s^j, \sigma_a^j, g, T^{12}, T^{21}), \{x \in \mathbb{R}^n\} \hspace{1cm} (6)$$

2.3 Model Analysis

In order to solve the optimizing problem posed in equation (6) above, we need to choose a optimizing method which have both robustness and fast speed to converge. The vector \(x\) as shown in (6) has large number of parameters; it is unreasonable to attempt optimizing such a large vector. It will both cost too much of time and consume much of the memory space.

Let’s revisit the equation (3):

$$L_r(\theta_r, \phi_r) = \zeta T^{12} T^{21} \frac{p(\phi, g)}{\cos \theta_i} \cos \theta_i \left(1 - e^{-\tau_d \frac{1}{\cos \theta_i + \cos \theta_r}}\right)L_i(\theta_i, \phi_i) + \rho \cos \theta_i$$

where

$$\zeta = \frac{\delta_s}{\delta_t}$$

and

$$\tau_d = \delta_t d$$

now if we set:

$$\delta_q = \frac{L_i \delta_s T^{21} T^{12}}{16 \pi^2} \hspace{1cm} (7)$$

and parameters like \(d\) and \(g\) have certain value, so set them be assigned a fixed value. Parameter \(\delta_t\) also have very small variance, so set \(\delta_t\) as constant value. After this, the \(x\) vector is separated to each pixel as follows:
\[ x_j = (n_{x}^{j}, n_{y}^{j}, n_{z}^{j}, \rho^{j}, \delta_{q}^{j}) \]  

(8)

The optimization problem can now be reduced as follows:

\[ \arg \min_{x} E(x_j), \text{where} \quad E(x_j) = \sum_{k} (L_t^{k} - l^k)^2 \]  

(9)

2.4 Experiments and Results

The primary input data consists of seven images acquired with different illumination, for each pixel, there are four independent parameters \((n_{x}^{j}, n_{y}^{j}, n_{z}^{j})\) are regarded as two independent parameters). The parameters are initialized as follows:

a) The parameters \(d, g, \delta_t\) can be fixed as 0.12mm, 0.81, 34.5mm\(^{-1}\), respectively using earlier study in cited references.

b) For \(n_{x}^{j}, n_{y}^{j}, n_{z}^{j}, \rho^{j}\), their initial values are calculated through the least square method with the original pixel data;

c) \(\delta_{q}^{j}\) are initialized to 0.

Part of the original input fingerprint images are shown in Figure 2. There are several methods in the literature to solve the optimization problems.

Figure 2: Seven fingerprint images of a subject acquired under different illumination.
(A) Solutions for Nonlinear optimization Problem

In order to solve nonlinear optimization problem (with constrained parameters) posed in previous section, we firstly consider solutions that are easily available in the Matlab environment (*function: fmincon.m*). The n vector has a nonlinear constraint of:

\[ n_x^2 + n_y^2 + n_z^2 = 1 \]  \hspace{1cm} (10)

Other constraints :

\[ \rho > 0, \delta_q > 0 \]  \hspace{1cm} (11)

We firstly use this implementation to solve the nonlinear optimization. Most of the recovered pixels were satisfactory but for some pixels, this approach does not work. The program just keeps on running and cannot converge or stop.

**Figure 3:** The results from the 3D fingerprint reconstruction: (a) HK model based reconstruction (using optimization function *fmincon*), (b) using Lambertian model.
This problem may caused by the complexity of the nonlinear constraint. So, if we are able to remove this nonlinear constraint, the function is more likely to solve the problem. Convert the coordinates to Polar Coordinates \((\theta, \varphi)\), we have:

\[
\begin{align*}
\theta &= \arcsin(n_z), \\
\varphi &= \arctan\left(\frac{n_y}{n_x}\right)
\end{align*}
\]

In this expression, the nonlinear constraint is eliminated. We rerun the program and now can get the results as also shown in figure 3. The results from the HK model have much noise. However, the reconstruction results from the Lambertian model are closer approximation to real finger surface and are also smooth.

**(B) Newton’s Method with Linear Search**

Newton’s method is one of the best alternatives to solve optimization problems. Its iteration function is like:

\[
x_{n+1} = x_n - \gamma [Hf(x_n)]^{-1} \nabla f(x_n)
\]

where \(Hf(x)\) is the Hessian matrix, \(\nabla f(x)\) is the gradient, and \(\gamma\) is the step size. In order to find the best step size, linear search method is adopted to find the best point in the direction of \(Hf(x)^{-1} \nabla f(x)\). Best point means \(E(x)\) in(9) reaches minimum value along this direction. Sample results from 3D fingerprint reconstruction using this method are shown in figure 4-5.
Figure 4: Reconstruction results from the two models. Each row belongs to a same finger.

For shadowed areas, the HK model generally resulted in poor results.

Figure 5: Reconstruction results from the shadowed areas; (a) result using HK model, (b) result using Lambertian model.

The results from Lambertian model and HK model are very similar. In fact, for many pixels which have good condition, the result of Lambertian model and HK model are the same. The $L_r$ in (3) is comprised with two parts. One is the Lambertian part, another one, let’s call HK part. The Lambertian part is dominant in the results. And for many pixels, the HK part is close to zero. Data is shown in Diag.1.

Table 1: Proportion of pixels contributed in 3D fingerprint reconstruction from HK part.

<table>
<thead>
<tr>
<th></th>
<th>Fingerprint 1</th>
<th>Fingerprint 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of pixels</td>
<td>350325</td>
<td>60996</td>
</tr>
<tr>
<td>pixels that has $\delta_q&lt;100$</td>
<td>340055</td>
<td>60996</td>
</tr>
<tr>
<td>pixels that has $\delta_q&gt;1000$</td>
<td>764</td>
<td>0</td>
</tr>
</tbody>
</table>

The fingerprint 1 and 2 are the images shown in figure 3. $\delta_q$ is the coefficient of the HK part. If $\delta_q=1000$, about 20% of the pixel’s value is determined by the HK part.

Another indicator of the performance is the energy, i.e. the $E(x)$ in (9), related
data is shown in table 2. Results of HK model have smaller energy but it doesn’t mean smaller energy is equal to a better 3D reconstruction result. For point with bad conditions (like specular points, shadowed points, etc.), Lambertian model usually has a larger energy than HK model but it’s more close to the real value.

**Table 2:** Total energy from the two reconstruction methods for two fingerprints.

<table>
<thead>
<tr>
<th></th>
<th>Fingerprint 1</th>
<th>Fingerprint 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lambertian model</td>
<td>$8.3246 \times 10^9$</td>
<td>$2.7087 \times 10^7$</td>
</tr>
<tr>
<td>HK model</td>
<td>$8.3237 \times 10^9$</td>
<td>$2.7068 \times 10^7$</td>
</tr>
</tbody>
</table>

**Results from blurry images**

If the images are blurry like the sample image in figure 6, we can also get the results.

![Reconstruction from blurry images](image)

(a) Original image, the one below is less blurry, (b) HK model results, (c) Lambertian model results.

**Figure 6:** Reconstruction from blurry images: (a) Original image, the one below is less blurry, (b) HK model results, (c) Lambertian model results.

**Reconstruction with Smaller Number of Images**

Since there are seven equations with less than five unknown parameters, we can
achieve 3D reconstruction less images. The results from using four or five images for reconstruction, with the Lambertian model, are like those shown in figure 7.

![Figure 7](image)

**Figure 7**: Reconstruction with less images. (a) with 4 images, there are some odd points. (b) with 5 images. It is more accurate but still some odd points.

The HK model has more parameters, *i.e.*, four in our method. Therefore using less images will make it hard to get the best parameters. Typical result of reconstruction with five images by HK model is illustrated in figure 8.

![Figure 8](image)

**Figure 8**: 3D fingerprint reconstruction results by using 5 images using the HK model. The results are poor in the shadowed areas.

**Improving Lambertian Model Results**

Some pixels in the acquired images are not available among some of the LED illumination(s). It is possible to achieve improved or more accurate result after filtering the shadowed value of such/certain pixels. On the other hand, specular
reflection will also do harm to the accuracy of Lambertian model. Specular value is also eliminated before the computing the results such as those shown in figure 9.

Figure 9: Improved Lambertian method results with shadowed and specular value elimination; (b) has larger shadow value threshold than used for (a). However the shadowed areas in the above images show poor results.

2.5 Revisiting HK Model and Newton’s Method with less Constraints
The cross section of sub layer scattering in above figure 10 illustrates more accurate details (layer 2-3) than those in figure 1 and allows us to revisit the model. The key objective is to once again explore the poor results from HK model in previous section. In HK model [4], the scattered intensity from the sub-layer is defined as follow:

\[
L_{r,v} = \frac{\sigma_q}{\sigma_t} \left( 1 - e^{-\sigma_t d \left( \frac{\cos \theta_i + \cos \theta_f}{\cos \theta_i \cos \theta_f} \right)} \right) + \rho \cos \theta_i = L_{r,v}^{(1)} + \rho \cos \theta_i
\]

where \( \rho \cos \theta_i \) is the higher order terms and the rest is the first order term \( L_{r,v}^{(1)} \)

\[
\sigma_q = \frac{L_i \sigma_s T_{21} T_{12}}{4\pi}, \quad \sigma_t = \sigma_s + \sigma_a, \quad \zeta = \frac{\sigma_s}{\sigma_t}.
\]

\[
L_r = L_{r,s} + L_{r,v}
\]

Assuming \( L_{r,s} \) is removed by rejecting specular pixel intensity, the observed intensity \( L_r \) depends on \( L_{r,v} \).

**Optimization problem:**

\[
\arg \min E(x_j), \quad E(x_j) = \sum_{k=1}^{M} (L_{k} - I_k)^2,
\]

where \( M \) is the number of valid input pixel intensity. Set \( \{d, \sigma_s, \sigma_a, g\} = \{0.12 \text{mm}, 30 \text{mm}^{-1}, 4.5 \text{mm}^{-1}, 0.81\} \) [4]. We can use details in [12] to select \( \sigma_s \) and \( \sigma_a \) as 6mm\(^{-1}\) and 8.5mm\(^{-1}\) for blue light at 470nm. The depth of Epidermis is around 0.325mm [13]. Let \( \{d, \sigma_s, \sigma_a, g\} = \{0.325 \text{mm}, 6 \text{mm}^{-1}, 8.5 \text{mm}^{-1}, 0.81\} \) be the new set of fixed parameter. Assuming \( T_{12} \) and \( T_{21} \) are random parameters, \( \sigma_q \) will be proportional to \( T_{12} \) and \( T_{21} \). The set of unknown parameter vector is \( x_j = (n_x^j, n_y^j, n_z^j, \rho^j, \sigma_q^j) \) and it is transformed to spherical coordinate to become \( x_j = (\theta^j, \phi^j, \rho^j, \sigma_q^j) \), where \( \theta = \arcsin(n_x) \) and \( \phi = \arctan(n_y/n_z) \).

**Constraints:** The constraints problem has not been imposed on the Newton method. The method explored in this section is based on the unconstrained Newton method with extra function to keep \( \rho \) and \( \sigma_q \) to be positive at the end of each iterations.

- \( \rho > 0, \sigma_q > 0 \) (\( \sigma_s, T_{12}, T_{21}, L_i \) are positive)
- \( 0 < \theta < 90 \) (all pixel surface are visible to camera)
- If \( \phi > 360 \), \( \phi = \phi - 360 \)
- If \( \phi < 0 \), \( \phi = \phi + 360 \) (repetition)

We use \( \cos \theta_i = \vec{n} \cdot \vec{n}, \cos \theta_f = \vec{n} \cdot \vec{n} \) and \( \cos \phi = -\vec{n} \cdot \vec{n} \) for the backward scattering. \( T_{12} \) and \( T_{21} \) is the transmission intensity coefficient which can be computed by the Fresnel coefficient. Let \( n_1 \) and \( n_2 \) be the refractive indices of medium 1 and medium 2. If the light perpendicularly strikes on the surface of the medium 2 from medium1, then the specular reflectance \( R_0 \) is
where \( n = 1.0 \) and 1.38 for air and Epidermis[6]. Using Schlick’s approximation [16]-[17], the Fresnel reflection factor \( R \) is computed by

\[
R = R_0 + (1 - R_0) (1 - \cos \theta_i)^5 \quad \text{for } n_1 < n_2, \\
R = R_0 + (1 - R_0) (1 - \cos \theta_r)^5 \quad \text{for } n_1 > n_2.
\]

The reflected energy coefficient \( R_p \) is defined by:

\[
R_p = |R|^2 \quad \text{(Conservation of energy)}
\]

The product of \( T^{12} \) and \( T^{21} \) will be

\[
T^{12} = \frac{n_1^2}{n_2^2} (1 - R_{p12}) \\
T^{21} = \frac{n_2^2}{n_1^2} (1 - R_{p21})
\]

\[
T^{12}T^{21} = (1 - R_{p12})(1 - R_{p21}) = (1 - (R_0 + (1 - R_0) (1 - \cos \theta_i)^5)^2)(1 - (R_0 + (1 - R_0) (1 - \cos \theta_r)^5)^2)
\]

Then the unknown vector is \( x_j = (\theta^j, \phi^j, \rho^j, L_1^j) \) for each pixel \( j \) in the input image.

**Experiments:**

<table>
<thead>
<tr>
<th>Parameter set</th>
<th>( d ) (mm)</th>
<th>( \sigma_s ) (mm(^{-1}))</th>
<th>( \sigma_a ) (mm(^{-1}))</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.12</td>
<td>30</td>
<td>4.5</td>
<td>0.81</td>
</tr>
<tr>
<td>B</td>
<td>0.325</td>
<td>6</td>
<td>8.5</td>
<td>0.81</td>
</tr>
</tbody>
</table>

**Table 3:** parameter values for set A and set B

<table>
<thead>
<tr>
<th>Model 1</th>
<th>( x_j = (\theta^j, \phi^j, \rho^j, L_1^j) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 2</td>
<td>( x_j = (\theta^j, \phi^j, \rho^j, L_1^j) )</td>
</tr>
</tbody>
</table>

Input resolution 300 \( \times \) 300 image, the output is 294 \( \times \) 294 resolution.

**Figure 11:** input image with LED1 (left), the value of \( \rho \) in HK model (right).
The value of $\rho$ and $L_i$ supposes to be close to a constant with some variation on the skin. The zero value and high value outlier of $\rho$ and $L_i$ exist in the current optimization with Newton’s method due to exceed the number of loop or non smooth derivative approximation (see figure 11). If the derivative of $\varphi$ is too steep and overshoot the new normal vector $\mathbf{n}_{t+1}$ such that $\theta_i > 90$ for all LED orients, $L_i$ will becomes 0 and the derivative of all direction of $(\theta_j, \phi_j, \rho_j, \sigma_q)$ will be 0. At the outlier point, the value of $\rho$ and $L_i$ will be close to zero or very high. $\varphi$ will be far away from the Lambertian model initialized value. Using Poisson Solver to reconstruct the surface from vector field, the reconstructed surface is totally distorted.

The value of $\rho$ is always smaller in the bottom of the valley that other region. It shows that the surrounding geometry can affect the model. In figure 10, if the incident light enter the skin at location P, the first order of $L_{sv}$ should exist the skin which are not far away from location P. However, the higher order terms of $L_{sv}$ can travel more distance in the layer 2 and exist at the adjacent locations P+1 or P-1. Measuring $L_{sv}$ at location P is the acuminate result of $L_{sv}(P) + L_{sv}^{(1)}(P) + \text{sum}(L_{sv}^{(n)}(P-1,P,P+1))$ assuming the high order subsurface scattering travel distance is not more than distance of 1 pixel of the capturing device. If the geometry of adjacent locations is alike (normal vectors are close), we can let $L_{sv}^{(n)}(P-1)=L_{sv}^{(n)}(P)$ and ignore the calculation of $L_{sv}^{(n)}$ in adjacent locations as the experiment result of current optimization.

![Histogram of p from Model 1 Par. B](image)

**Figure 12**: The distribution of value of $p$ with model 1 and parameter set B. Note: the value of 255 is including the value bigger than 255, since the data is stored with 8 bit grayscale bmp for quick visual inspection. The count is peaked at value 38 with 45 counts. Half height values are around 45 and 33, and the half width is around (5-7)*2.
3. Conclusions

Compared to the HK model which needs a lot of iterations, Lambertian model for 3D fingerprint reconstruction only uses linear matrix computing, which makes it more stable in some regions with bad conditions. HK model uses nonlinear optimizing method which often gives a local optimal solution. The Newton’s method for optimization cannot guarantee to achieve best or optimal solution; this can be considered as mathematical limitation of HK model.

In this technical report, we have attempted to reconstruct 3D fingerprint using linear reflection model from Lambertian approach and compared with those from non-Lambertian approach. Our key objective in this work has been to compare these two approaches to ascertain its usability for online 3D fingerprint reconstruction. Our experimental results reported in this technical report suggests that HK model for 3D fingerprint reconstruction does not generate more accurate results, as compared to those from Lambertian model. This study should complement the effort reported in [11] which fails to provide any comparison to underline on any possible advantages. On the other hand, the Lambertian model has significantly lower complexity and generates more accurate results that can justify using this approach for its use in 3D fingerprint reconstruction.

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References